Assessment Schedule – 2015

Mathematics and Statistics: Apply algebraic methods in solving problems (91261)

Evidence

One	Expected Coverage	Understanding (u)	Relational thinking (r)	Abstract thinking (t)
(a)(i)	$2^{x} = 1024$ $x = 10$	Equation solved.		
(a)(ii)	$3w + 1 = 4^2$ 3w = 15 and $w = 5$	Equation solved.		
(a)(iii)	$x^{2} = 4x + 12$ $x^{2} - 4x - 12 = 0$ (x - 6)(x + 2) = 0 x = 6 or -2 But base must be positive x = 6 is the only solution	Sets up a quadratic equation.	Solved problem using quadratic, but gives both values.	Gives only valid solution with justification.
(b)	$2x \log a = (x+1) \log b$ $x(2 \log a - \log b) = \log b$ $x = \frac{\log b}{2 \log a - \log b}$	Takes logs of both sides and multiplies by indices.	Takes logs of both sides and rearranges.	Correctly solved.
(c)(i)	$P = A \times (1.03)^{t}$ Beginning of 1999, $t = 0$, $t = 16$, $P = 350\ 000$ $350\ 000 = A\ (1.03)^{16}$ $A = 218\ 108$ So price was \$218\ 108 initially.	Sets up model correctly.	Answers question in context correctly.	
(c)(ii)	$2 18 100 (1.03)^{t} = 200\ 000\ (1.035)^{t}$ $\frac{218\ 100}{200\ 000} = \left(\frac{1.035}{1.030}\right)^{t}$ $1.0905 = 1.004854369^{t}$ $t = \frac{\log 1.0905}{\log 1.004854369}$ $t = 17.89$ In 2016.	Set up correct equation.	Solved for <i>t</i> .	Correct year identified.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Attempt at one question	l of u	2 of u	3 of u	l of r	2 of r	1 of t	2 of t

Тwo	Expected Coverage	Understanding (u)	Relational thinking (r)	Abstract thinking (t)
(a)	$\frac{(x+4)(2x-1)}{2(x^2-16)} = \frac{(x+4)(2x-1)}{2(x-4)(x+4)} = \frac{(2x-1)}{2(x-4)}$ Provided $x \neq \pm 4$	Factorised and simplified with one error.	Correctly simplified.	
(b)	$a^{7} = \left(y^{\frac{3}{4}}\right)^{7}$ $= y^{\frac{21}{4}}$	Correct expression.		
(c)	Let $x = u^{\frac{1}{3}}$ $2x^2 + 7x - 4 = 0$ (2x - 1)(x + 4) = 0 $x = \frac{1}{2}$ or $x = -4$ $u^{\frac{1}{3}} = \frac{1}{2}$ so $u = \frac{1}{2^3} = \frac{1}{8}$ OR $u^{\frac{1}{3}} = -4$ so $u = (-4)^3 = -64$	CAO or rewrites as quadratic.	Solves for <i>x</i> .	Solves completely with both solutions.
(d)(i)	Let x be the length and w the width. Then the perimeter is $2x + 2w$. Area $xw = 50$ So $w = \frac{50}{x}$ or $2w = \frac{100}{x}$ So perimeter = $2x + \frac{100}{x}$	Shows relationship.		
(d)(ii)	$2x + \frac{100}{x} = 33$ $2x^{2} - 33x + 100 = 0$ (2x - 25)(x - 4) = 0 x = 12.5 or x = 4 m So the dimensions of the garden are 4 m and 12.5 m.	Forms a quadratic equation	Solved for <i>x</i> , or consistently solved from incorrect quadratic.	Correctly solved and dimensions given.

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(e)	David's speed is $x \text{ km / h}$ Sione's speed is $(x + 4) \text{ km / h}$	Sets up equation correctly and solves with an error.	Correctly sets up equation and solves correctly.
	Difference in time is half an hour.		
	$0.5 = \frac{150}{x} - \frac{150}{x+4}$		
	$0.5 = \frac{150(x+4) - 150x}{x(x+4)}$		
	0.5x(x+4) = 600		
	$x^2 + 4x - 1200 = 0$		
	x = 32.70 km / hr		

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Attempt at one question	l of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Three	Expected Coverage	Understanding u	Relational thinking r	Abstract Thinking t
(a)(i)	$\left(\frac{a^{10}}{4a^5}\right)^{-2} = \left(\frac{4a^5}{a^{10}}\right)^2$ $= \left(\frac{4}{a^5}\right)^2 = \frac{16}{a^{10}}$	Evidence of correctly simplifying the negative or index or square or numerator or denominator correct	Algebraic expression simplified.	
(ii)	$\sqrt[5]{\left(\frac{32}{x^5}\right)^3} = \left(\frac{32}{x^5}\right)^{\frac{3}{5}} = \frac{32^{\frac{3}{5}}}{\left(x^5\right)^{\frac{3}{5}}} = \frac{\left(\sqrt[5]{32}\right)^3}{x^3} = \frac{8}{x^3}$	Numerator or denominator correct.	Algebraic expression simplified.	
(b)	$\frac{1}{t(t-1)} - \frac{t-1}{t(t-1)} - \frac{3t}{t(t-1)} = 0$ $\frac{1-t+1-3t}{t(t-1)} = 0$ $\frac{2-4t}{t(t-1)} = 0$ $\frac{2(1-2t)}{t(t-1)} = 0$ $t = \frac{1}{2}$	Partially solved by rewriting over correct common denominator.	Correctly solved.	
(c)	Never touch the <i>x</i> -axis means $\Delta < 0$ $(3k-1)^2 - 4(2k+10) < 0$ $9k^2 - 6k + 1 - 8k - 40 < 0$ $9k^2 - 14k - 39 < 0$ If $9k^2 - 14k - 39 = 0$ Then $(9k + 13)(k - 3) = 0$ and $k = 3$ or $-\frac{13}{9}(-1.44)$ So if the graph does not cut the <i>x</i> -axis, then $-\frac{13}{9} < k < 3$	$\Delta < 0$	Correct solutions for <i>k</i> .	Problem solved with correct inequality.

(d)	If both roots real, so $\Delta > 0$			
	ie $[-(m+2)]^2 - 4m \times 2 > 0$	$\Delta > 0$		
	So $m^2 - 4m + 4 > 0$			
	i.e. $(m-2)^2 > 0$			
	So <i>m</i> can be any real number but $m \neq 2$, as any number squared except zero is always positive.			
	Using the quadratic formula or otherwise, the roots are			
	$\frac{m+2\pm\sqrt{(m-2)^2}}{2m}$		$(m-2)^2 > 0$ and $m \neq 2$	
	$=\frac{m+2\pm(m-2)}{2m}$			
	$x_1 = \frac{2m}{2m} = 1$ provided $m \neq 0$		OR	
	or $x_2 = \frac{4}{2m} = \frac{2}{m}$ provided $m \neq 0$		BOTH roots found.	
	So to fill all conditions including both roots are positive real, we have $m > 0$ $m \neq 2$ with roots 1 and $\frac{2}{m}$.			Problem solved correctly.

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Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 14	15 – 19	20 – 24