# Assessment Schedule – 2019

# Mathematics and Statistics: Apply algebraic methods in solving problems (91261)

## Assessment Criteria

| Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|---|
| *Apply algebraic methods in solving problems* involves:<br><br>• selecting and using methods<br>• demonstrating knowledge of algebraic concepts and terms<br>• communicating using appropriate representations. | *Apply algebraic methods, using relational thinking, in solving problems* must involve one or more of:<br>• selecting and carrying out a logical sequence of steps<br>• connecting different concepts or representations<br>• demonstrating understanding of concepts<br>• forming and using a model<br>and also relating findings to a context, or communicating thinking using appropriate mathematical statements. | *Apply algebraic methods using extended abstract thinking, in solving problems* involves one or more of:<br>• devising a strategy to investigate or solve a problem<br>• identifying relevant concepts in context<br>• developing a chain of logical reasoning, or proof<br>• forming a generalisation<br>and also using correct mathematical statements or communicating mathematical insight. |

## Evidence

| ONE | Expected Coverage | Achievement (u) | Merit (r) | Excellence (t) |
|---|---|---|---|---|
| (a)(i) | $3x^2 - 7x - 6 = (3x + 2)(x - 3) = 0$<br><br>$x = \frac{-2}{3}$, 3 or equivalent. | Correct solutions. | | |
| (a)(ii) | $5x^2 - 4x - 3 = 0$<br><br>$x = 1.27, -0.47$ or equivalent. | Correct solutions. | | |
| (b) | $5.05 = 0.02t^2 - 0.6t + 9.18$<br><br>$0.02t^2 - 0.6t + 4.13 = 0$<br><br>$t = 10.7, 19.3$ Hence $t = 10.7$ months (cannot be 19.3) | Quadratic equation set equal to 0. | Correct answer. | |
| (c) | Discriminant $\Delta = b^2 - 4ac = 0$<br><br>$(m + 1)^2 - 4(2m - 1)(m - 4) = 0$<br><br>$7m^2 - 38m + 15 = 0$<br><br>$(7m - 3)(m - 5) = 0$<br><br>$m = 5$<br><br>($\frac{3}{7}$ eliminated as $m - 4$ must be at least 0 since the expression is a perfect square) | Sets up discriminant equal to 0. | Sets up quadratic equation equal to 0. | Correct solution incorporating clear explanation of the rejection of $m = \frac{3}{7}$. |

| | | | | |
|---|---|---|---|---|
| (d) | $p^2x^2 + 4px - 12 = (px + 6)(px - 2) = 0$<br><br>$x = \frac{-6}{p}, \frac{2}{p}$<br><br>Hence $\frac{2}{p} - \frac{-6}{p} = \frac{8}{p}$. | Factorises correctly or finds the difference between roots consistently. | The correct roots are found. | Finds the difference between the correct roots. |
| (e) | $y = x^2 - bx - ax + ab - c^2$<br><br>$= x^2 - (a + b)x + (ab - c^2)$<br><br>$\Delta = (-(a + b))^2 - 4 \times 1 \times (ab - c^2)$<br><br>$= (a^2 - 2ab + b^2) + 4c^2$<br><br>$= (a - b)^2 + 4c^2$<br><br>As $(a - b)^2 \geq 0$ and $c^2 > 0$ then $\Delta > 0$ and hence there are two real distinct roots (and two distinct points where the graph crosses the $x$–axis).<br><br>Graphical argument:<br>"Since $f(x) = (x - a)(x - b)$ is a positive parabola which clearly has 2 roots and its graph crosses the $x$-axis at 2 distinct points, the function $g(x) = f(x) - c^2$, which must be lower since $c^2$ must be positive, must also have 2 distinct roots (which would be further apart than a and b)." | Function is set up so that the discriminant can be found. | Finds discriminant in factored form. | Explains why the discriminant is greater than 0 and makes a correct conclusion.<br><br>OR graphical argument is fully described. |

| N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
|---|---|---|---|---|---|---|---|
| A valid attempt at one question. | 1 of u | 2 of u | 3 of u | 1 of r | 2 of r | 1 of t | 2 of t |

**N0 =** No response; no relevant evidence.

| TWO | Expected Coverage | Achievement (u) | Merit (r) | Excellence (t) |
|---|---|---|---|---|
| (a)(i) | $9^{0.5}a^{1}b^{-2}$<br>$=\frac{3a}{b^2}$ | Correct answer. | | |
| (a)(ii) | $\left(\frac{3b^4}{2a}\right)^2=\frac{9b^8}{4a^2}$ | Correct answer. | | |
| (b) | $\frac{2c+1}{(c+3)(c-3)}+\frac{c-1}{(c-3)(c-1)}$<br>$=\frac{(2c+1)(c-1)+(c-2)(c+3)}{(c+3)(c-3)(c-1)}$<br>$=\frac{3c^2-7}{(c+3)(c-3)(c-1)}$ | Cross-arrangement to a single fraction. | Final simplification. | |
| (c) | $fm - 2gm - 6gn + 3fn$<br>$= m(f - 2g) + 3n(f - 2g)$<br>$= (m + 3n)(f - 2g)$ | Pairs factored. | Complete factorisation. | |
| (d) | Small rectangle: Area = $y^2 - 8y = 9$<br>$y^2 - 8y - 9 = (y - 9)(y + 1) = 0 \Rightarrow y = 9$<br>Since $x = 2y - 6$<br>Large rectangle: Area = $(2y - 6)(2y - 10)$<br>$= 4y^2 - 32y + 60$<br>Hence Area = 96 cm$^2$ | $y$ found. | | Area found. |
| (e) | Roots are $\frac{-p+\sqrt{p^2-4q}}{2}$ and $\frac{-p-\sqrt{p^2-4q}}{2}$<br>$-p+\sqrt{p^2-4q}=n(-p-\sqrt{p^2-4q})$<br>$\sqrt{p^2-4q}=\frac{p(1-n)}{(1+n)}$<br>$qn^2+(2q-p^2)n+q=0$ | One root $n$ times the other. | Successfully squares both sides. | Finds equation with correct algebraic working. |

| N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
|---|---|---|---|---|---|---|---|
| A valid attempt at one question | 1 of u | 2 of u | 3 of u | 1 of r | 2 of r | 1 of t | 2 of t |

**N0** = No response; no relevant evidence.

| THREE | Expected Coverage | Achievement (u) | Merit (r) | Excellence (t) |
|---|---|---|---|---|
| (a) | $\log_5(m) = 3$<br>$\Leftrightarrow\ 5^3 = m \Leftrightarrow m = 125$ | Correct answer. | | |
| (b) | $\log 6 - 2\log y$<br>$= \log\left(\frac{6}{y^2}\right)$ | Correct answer. | | |
| (c) | $\frac{3^{2n-1}+3^{2n+1}}{3^{2n}-3^{2n-4}} = \frac{3^{2n-4}(3^3+3^5)}{3^{2n-4}(3^4-1)}$<br>$= \frac{27+243}{81-1} = \frac{27}{8}$ or equivalent. | Correct answer only. | Finds common factor of numerator and denominator. | Correct answer. |
| (d)(i) | $3N_0 = N_0(1.053)^t$<br>$3 = (1.053)^t$<br>$\log(3) = t\log(1.053)$<br>$t = \frac{\log(3)}{\log(1.053)}$ = 21.27 weeks | Taking log of both sides and $t$ as a factor. | Correct answer. | |
| (d)(ii) | $\frac{4250}{2500} = (1+\frac{r}{100})^{10}$<br>$1+\frac{r}{100} = \sqrt[10]{1.7} = 1.0545$<br>Hence r = 5.45 and rate of change is 5.45%. | Sets up correct equation. | Finds $1+\frac{r}{100}$ . | Percentage rate of change found. |
| (e) | $k = \frac{227-67}{(640)^2} = 0.000390625$<br>$h = 0.000390625\,(x-640)^2 + 67 = 100$<br>$0.000390625x^2 - 0.5x + 127 = 0$<br>$x$ = 349.3, 930.7 m<br>Required distance = 349.3 m | | Finds $k$<br>OR<br>finds two consistent solutions from calculated $k$. | Correct answer. |

| N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
|---|---|---|---|---|---|---|---|
| A valid attempt at one question | 1 of u | 2 of u | 3 of u | 1 of r | 2 of r | 1 of t | 2 of t |

**N0** = No response; no relevant evidence.

## Cut Scores

| Not Achieved | Achievement | Achievement with Merit | Achievement with Excellence |
|---|---|---|---|
| 0 – 8 | 9 – 14 | 15 – 19 | 20 – 24 |