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Level 2 Mathematics and Statistics 2020

91261 Apply algebraic methods in solving problems

9.30 a.m. Thursday 19 November 2020
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

You are required to show algebraic working in this paper. Guess-and-check methods or correct answer(s) only, will generally limit grades to Achievement.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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(a) Factorise $6x^2 + 13x - 15$.

(b) A function is defined as $f(x) = x^2 + 10x + 22$.
Express $f(x)$ in completed square form, i.e. $f(x) = (x + a)^2 + b$, where a and b are integers.

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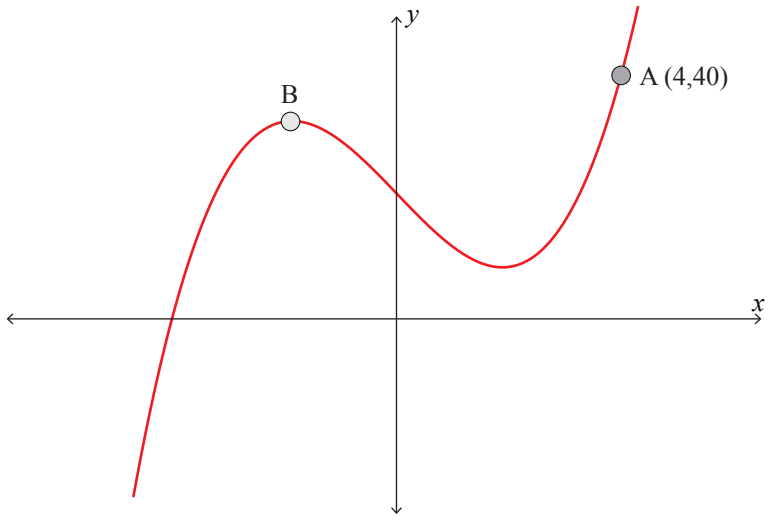
(c) In the 16th century, mathematicians were developing a formula to solve any cubic equation. They used expressions in the form of $y = x^3 - 12Px + R$, where P and R are positive constants.

(i) The graph of $y = x^3 - 12Px + R$, for some values of P and R, passes through the point A (4,40) and is sketched below.



Find an expression for P in terms of R.

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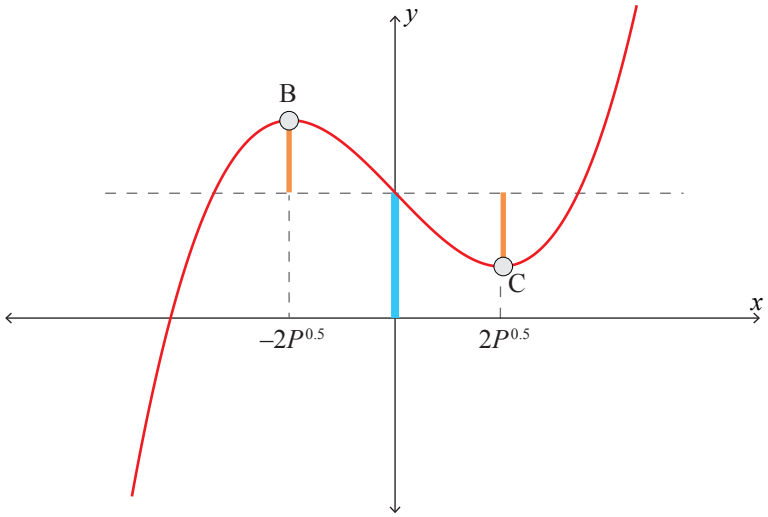
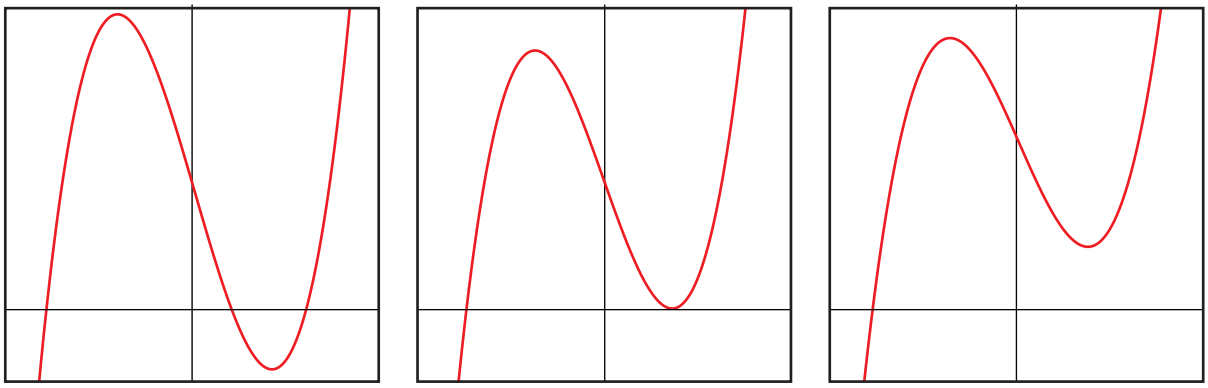
(ii) At point B it is true that $3x^2 - 12P = 0$.

Using algebra, show that $x = -2P^{0.5}$ at B.

(iii) Consider again the curve with the equation $y = x^3 - 12Px + R$. As the values of P and R vary, the shape of the curve changes, and the lengths of the orange lines and of the central blue line (below) vary. However, by symmetry, the two orange lines remain the same length as each other.



Some examples of the graphs obtained from various values of P and R are illustrated below.



For some combinations of P and R, the curve can intersect the x-axis three times. This will happen if each orange line is longer than the blue line.

Find the condition that P and R must satisfy for the graph to have three distinct x-intercepts.
Write your answer in a form where the exponents of P and R are both positive integers.

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QUESTION TWO

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(a) Write as a single logarithm in simplest form: $\log(9y) + \log(4) - \log(3y)$.

(b) Solve each of the following equations:

(i) $\log_x(36) = 2$.

(ii) $\log_5(x) + \log_5(2x) = 4$.

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(c) Solve the equation: $\frac{5x+4}{x+4}-\frac{3x-4}{2x+1}=2$.

(d) Consider two parabolas:

- Parabola One given by $y = ax^2 + bx + c$ and
- Parabola Two given by $y = dx^2 + ex + c$, where a, b, c, d, and e are constants.

Use algebra to determine the restrictions on the values of a, b, c, d, and e that would ensure that the parabolas meet at two distinct points.

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QUESTION THREE

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(a) Solve the equation $3^{4x} = 30$.

(b) Consider the function $W = (x+2)^{\frac{2}{5}}$, where x is a **whole number**.

(i) Make x the subject of the formula $W = (x+2)^{\frac{2}{5}}$.

(ii) For what values of x will the function have values less than 20?

Please turn over ➤

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(c) Zahra sells zips. Zahra notices that the higher the price of a zip, the fewer zips are sold. As an experiment, Zahra increases the price of a zip by \$2 each day (starting at \$7) and keeps a record of how many zips are sold each day. She does this for 6 days and finds that the number of zips sold each day started at 98 and is dropping by 3 each day.

The total amount of money Zahra received each day for zips, the turnover, is also recorded in the table below.

Day, *d*	**1**	**2**	**3**	**4**	**5**	**6**
Price of a zip (\$) = $2d + 5$	7	9	11	13	15	17
Number of zips sold = $101 - 3d$	98	95	92	89	86	83
Turnover (\$)	686	855	1012	1157	1290	1411

(i) If all the patterns continue to be valid, is there any day on which the turnover is exactly \$445? Use algebra to justify your answer and explain your conclusions.

(ii) Zahra realises that not every whole number is a possible turnover value for a given day.

Using algebra, find at least three conditions a whole number k must satisfy for it to be a possible turnover for a given day.

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Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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