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Tick this box if you have NOT written in this booklet

Level 2 Mathematics and Statistics 2022

91261 Apply algebraic methods in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply algebraic methods in solving problems.	Apply algebraic methods, using relational thinking, in solving problems.	Apply algebraic methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae Sheet L2–MATHF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2 – 16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

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The examination continues on the following page.

QUESTION ONE

(a) Solve $\dfrac{2x-3}{x+4}-3=0$.

(b) (i) Factorise completely $6x^3y-15x^2\sqrt{y}$.

(ii) Simplify fully $\dfrac{6x^2-x-12}{3x^2-5x-12}$.

(c) A calendar can be presented in the following way, where each day is given a number from 1 to 365. This is the beginning of a year's calendar:

M	**T**	**W**	**TH**	**F**	**SA**	**SU**
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
32	33	34	35	36	37	38
39	40	41	42	43	44	45
46	47	48	49	50	51	52
53	54	55	56	57	58	59
60	61	62	63	64	65	66
67	68	69	70	71	72	73

Jo draws a 4-by-4 square on the calendar to check a claim that she heard:

"*the sums of the diagonally opposite corners are always the same, no matter where you make your square*". In other words, when you add the numbers in the orange corners, it is the same as when you add the numbers in the blue corners.

Jo wonders if the claim will still be true no matter where she starts the square, so she begins an investigation using algebra:

A			

(i) Use algebra to prove that, no matter where the 4-by-4 square is drawn on the calendar, **the sum of the orange corners must be the same as the sum of the blue corners**.

(ii) Jo wonders about the **products** of the diagonally opposite corners: could they be the same?

Use algebra to prove that it is **not** possible to draw any 4-by-4 square for which the products of the diagonally opposite corners are the same.

(d) Will it always be true that the **sum** of the orange corners must be the same as the sum of the blue corners, regardless of the size or shape of the rectangle Jo draws?

Use algebra to support your answer by considering an m-by-n rectangle drawn on the calendar below (where m and n are whole numbers greater than 1, and $m \neq n$).

You may wish to draw a diagram on the calendar, or beside it, to help explain your reasoning.

M	T	W	TH	F	SA	SU
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
32	33	34	35	36	37	38
39	40	41	42	43	44	45
46	47	48	49	50	51	52
53	54	55	56	57	58	59
60	61	62	63	64	65	66
67	68	69	70	71	72	73

QUESTION TWO

(a) A quadratic equation, $ax^2 + bx + c = 0$, has solutions of $\frac{1}{3}$ and $\frac{-2}{7}$.

Find the values of the integers a, b, and c.

a = ______ b = ______ c = ______

(b) (i) What is the discriminant of the equation $2x^2 - 12x + 7 = 0$?

(ii) Suppose $y = 2x^2 - 12x + k$, where k is a constant.

For what value of k will the equation $y = 0$ have exactly one solution?

(c) Solve $\sqrt{2x+3} = 3x$.

(d) Suppose $Q(x) = fx^2 + gx + h$, where f, g, and h are constants. The "reciprocal polynomial" of $Q(x)$ is defined as $Q^*(x) = hx^2 + gx + f$, where the coefficients are in the reverse order.

(i) Find the solutions of the equation $Q(x) = Q^*(x)$.

(ii) Suppose that $Q(x) = 0$ has 2 different roots, A and B.

The roots of $Q^*(x) = 0$ are multiples of A and of B, i.e. the roots are kA and kB for some constant k.

Find an expression for k in terms of f, g, and / or h.

QUESTION THREE

(a) (i) Simplify fully $\sqrt{49y^{36}}$.

(ii) Solve the equation $2^x = 2022$.

(b) Simplify the following expression fully, writing your answer as a single logarithm.

$$\log(3a)+2\log\left(\frac{a}{6}\right)$$

(c) Consider the equation $\log_2(x - a) - \log_2(x + a) = c$, where a and c are constants.

(i) Show that when x is made the subject of this equation, $x = a\dfrac{1+2^c}{1-2^c}$.

Ensure that you use correct mathematical statements in your reasoning.

Question Three continues on the following page.

(ii) The equation $\log_2(x - a) - \log_2(x + a) = c$, is only possible to solve for some values of a and for some values of c.

Explaining your reasoning clearly, describe which values of a and of c will make the equation possible to solve.

You may find it useful to recall that, when x is made the subject of this equation, $x = a\dfrac{1+2^c}{1-2^c}$.

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