Assessment Schedule – 2018

Mathematics and Statistics: Apply calculus methods in solving problems (91262)

Assessment Criteria

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$f'(x) = 3x^2 - 6$ $f'(4) = 3 \times 16 - 6$ $= 42$	Derivative AND gradient found.		
(b)	Let the width of the rectangle be w cm and the area be A cm^2. Then $A = 3w^2$ $A = 75$ means $w = 5$ $\frac{dA}{dw} = 6w$ so rate of change of area = 30 cm^2 / cm when $A = 75$.	Expression found for $\frac{dA}{dw}$.	Rate of change found.	
(c)	$f(x) = \frac{-3}{3}x^3 + \frac{12}{2}x^2 + c$ $f(0) = 5$, so c = 5 $f(x) = -x^3 + 6x^2 + 5$ Max and min are when $f'(x) = 0$ $-3x^2 + 12x = 0$ $-3x(x - 4) = 0$ so when $x = 0$ or $x = 4$ $x = 4$, $f(4) = 37$ so local max is 37.	$f(x)$ found. OR $f'(x) = 0$ and $x = 4$ found.	Local maximum found.	
(d)	$f'(x) = 2x^2 + 9x - 5$ increasing when $2x^2 + 9x - 5 > 0$ $(2x - 1)(x + 5) > 0$ $x < -5$ or $x > 0.5$	Derivative found and made more than 0 or equated to 0.	–5 and 0.5 found.	Restrictions on x found.

(e)	Area = $80 - 2 \times 0.5x(10 - x) - 2 \times 0.5x(8 - x)$ $= 80 - 18x + 2x^2$ $\frac{dA}{dx} = 4x - 18$ For min area, $\frac{dA}{dx} = 0$ so $x = 4.5$ for min area. Possible justifications: • A(x) is a parabola facing up, which only has a minimum. • $A'(4) = -2$, $A'(5) = 2$, so it is a minimum. • $A''(x) = 4 > 0$, so it is a minimum. Then area of parallelogram is $39.5\ cm^2$. Be aware of possible RAWW if area of only 2 triangles is used.	Area expression (mei) consistently differentiated.	x-value of minimum found.	Minimum area found and nature of this turning point justified correctly using a valid method.

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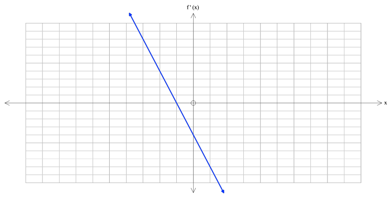
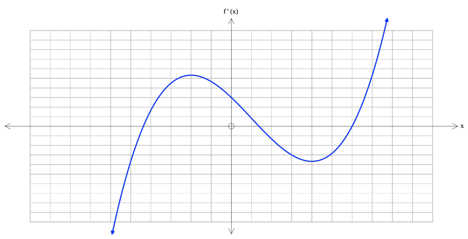
TWO (a)		Straight line with negative slope passing through x-axis at correct matching point ($x = -1$ if scales were integers).		
(b)	$\frac{dh}{dt} = 39.2 - 9.8t$ Max height when $\frac{dh}{dt} = 0$ So $t = 4$ and max height = 78.4 m.	Derivative found AND made equal to 0.	Time for max found, and max height found.	
(c)	$v(t) = 6t - 6t^2 + c$ $v(2) = 20$, so c = 32 $v(t) = 6t - 6t^2 + 32$ $s(t) = 3t^2 - 2t^3 + 32t + k$ $t = 0$, k = 0 $v(t) = 20$ m s^{-1} $t = 2$ and $s = 60$ m	Velocity equation found.	Distance equation found. Evidence of $t = 0$ means k = 0.	Distance in context when velocity is 20 m s^{-1} found
(d)		Graph is a positive cubic and goes through (0,3).	Graph is a positive cubic and goes through (0,3) and turning points match up with roots of $f'(x)$.	
(e)	$\frac{dy}{dx} = 3x^2 - 12x + k$ $\frac{dy}{dx} = 0$ when $x = 3$. so k = 9 $y = x^3 - 6x^2 + 9x - 5$ Turning points are when $\frac{dy}{dx} = 0$ $3(x-1)(x-3)=0$ so when $x = 1$ or $x = 3$ $x = 1, y = -1$ and when $x = 3, y = -5$ so using the y values, (1,–1) is a maximum and (3,–5) is a minimum. (OR using the graph of the function as positive cubic the max occurs to the left of a local minimum.)	Derivative found and set equal to 0.	Value for k found and x values of turning points found.	Problem solved finding the local max and min values with justification. (Accept any method.)

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$y = -x^5 + 6x + c$ when $x = 1$, $y = 7$ so $c = 2$ $y = -x^5 + 6x + 2$	Function found by anti-differentiating. $c = 2$ accepted. $f(x)$ not required.		
(b)	$\frac{dN}{dt} = 400 + 200t$ $\frac{dN}{dt} = 14\,400$ $400 + 200t = 14\,400$ $t = 70$ days	Rate of change of number of traders found, equated to 14 400 and time found.		
(c)	$R(p) = 1160p - 80p^2$ $R'(p) = 1160 - 160p$ Max revenue when $R'(p) = 0$ $1160 = 160p$ $p = 7.25$ so price of ticket is \$7.25 to maximise revenue at \$4205.	Finds $R'(p)$ and equates to 0.	Correct maximum revenue found.	

(d)	**Method One** (–6,64) on graph means $-\frac{1}{3}(6)^3 + -6k + 4 = 64$ so k = 2 $\frac{dy}{dx} = -3 \cdot \frac{1}{3}x^2 + 2$ $-x^2 + 2 = -7$ $x^2 = 9$ $x = 3 \text{ or } -3$ $x = 3, y = 1$ Tangent at (3,1) is $y = -7x + 22$ and (–6,64) is also on this line, so this is the correct point where tangent touches the graph. If $x = -3$, y = 7, then tangent at (–3,7) is $y = -7x - 14$ but (–6,64) is not on this line. **Method Two** Line through (–6,64) with gradient –7 is $y = -7x + 22$ (–6,64) on graph meaning $-\frac{1}{3}(6)^3 + -6k + 4 = 64$ so k = 2 $y = -7x + 22$ and $y = -\frac{1}{3}x^3 + 2x + 4$ intersect at (–6,64) and (3,1). need to check gradient at $x = 3$. Derivative of given function is $f'(x) = -3 \cdot \frac{1}{3}x^2 + k$ $f'(3) = -9 + 2$ = –7 as required so (3,1) is where tangent touches the graph.	Finds derivative and sets equal to –7 getting a quadratic OR Finds equation of line through (–6,64) with gradient of –7. And differentiates cubic.	Value of k found. AND Derivative found. AND Set equal to –7. OR Line with gradient –7 through (–6,64) found. AND k found. AND Derivative found.	T1: Correct point of (3,1) found. T2: Validity of (3,1) clearly and correctly justified.

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 14	15 – 19	20 – 24