

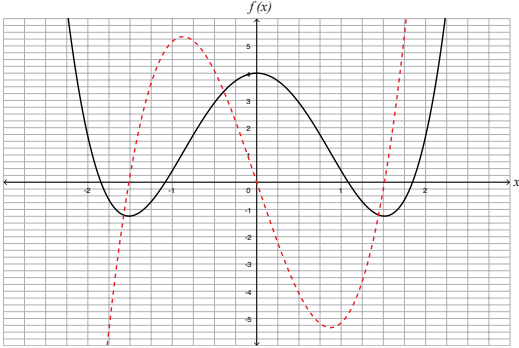
Assessment Schedule – 2021**Mathematics and Statistics: Apply calculus methods in solving problems (91262)****Evidence**

| Q ONE | Expected coverage | Achievement (u) | Merit (r) | Excellence (t) |
|---------|---|--|----------------------------------|----------------|
| (a) | $f'(x) = 12x^2 - 4x - 7$ $f'(3) = 12(3)^2 - 4(3) - 7$ $f'(3) = 89$ | Derivative found and gradient evaluated. | | |
| (b) | $f'(x) = \frac{3x^2}{2} + \frac{1}{2}$ $f'(2) = \frac{3(2)^2}{2} + \frac{1}{2}$ $f'(2) = 6.5$ $f(x) = \frac{x^3}{2} + \frac{x}{2}$ $f(2) = \frac{(2)^3}{2} + \frac{(2)}{2}$ $f(2) = 5$ Tangent at point (2,5) with a slope of 6.5 $(y - y_1) = m(x - x_1)$ $(y - 5) = 6.5(x - 2)$ $y = 6.5x - 8$ | Correct derivative. | Correct equation of the tangent. | |
| (c)(i) | $V(t) = 3520$ $3520 = -11(t)^2 + 528t$ $0 = -11(t)^2 + 528t - 3520$ $t = 8, 40$ $V'(t) = -22t + 528$ $V'(8) = -22(8) + 528$ $V'(8) = 352$ $V'(40) = -22(40) + 528$ $V'(40) = -352$ | Expression found for $\frac{dV}{dt}$. | Rates of change found. | |
| (c)(ii) | $V'(t) = -22t + 258$ $V' = 0$ $0 = -22t + 258$ $t = 24$ $V(24) = -11(24) + 528(24)$ $V(24) = 6336$ | Derivative found and set to 0. | Max daily viewers found. | |

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| (c)(iii) | $V' = 4.8t^2 - 260t + 2900$ $0 = 4.8t^2 - 260t + 2900$ $t = 15.71 \text{ or } 38.46$ $t = 15.71, V = 19\,678$ $t = 38.46, V = 10\,264 \text{ (or } 10\,263)$ Once this curve reaches $V = 10\,000$, it never again falls below $10\,000$ Increasing when: $t < 15.71$ and $t > 38.46$ Decreasing when: $15.71 < t < 38.46$ | t values of turning points found. | Coordinates of minimum point found. | Coordinates of both turning points found, statement regarding monetisation. T1: Excellence criteria satisfied with one aspect missing. T2: Justification from: Graph of function or gradient function, gradient on each side of the points, second derivative or substitution in to the function. |
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Evidence Statement

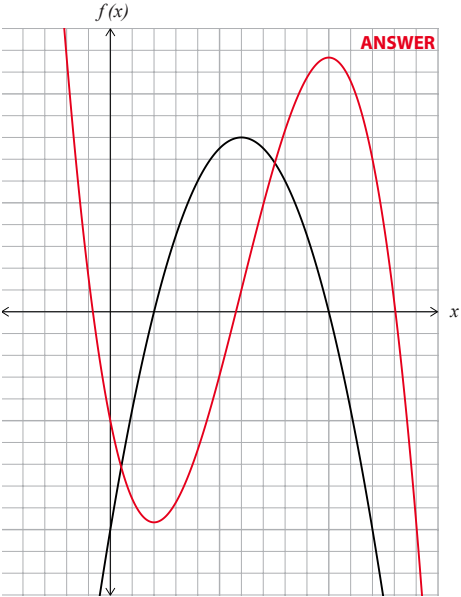
| NØ | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
|------------------------------------|--------------------------|--------|--------|--------|--------|--------|----|----|
| No response; no relevant evidence. | Attempt at one question. | 1 of u | 2 of u | 3 of u | 1 of r | 2 of r | T1 | T2 |

| Q TWO | Expected coverage | Achievement (u) | Merit (r) | Excellence (t) |
|----------|--|---|--|----------------|
| (a) |  <p>Red curve is solution, x-intercepts are at $-1.5, 0$ and 1.5</p> | <p>Correct shape and orientation of curve OR correct x-intercepts.</p> | <p>Correct shape and orientation of curve AND correct x-intercepts.</p> | |
| (b) | $f'(x) = 3 + 2cx - 6x^2$ $3 + 2c(2) - 6(2)^2 = -5$ $4c = 16$ $c = 4$ | <p>Derivative found and equated to -5.</p> | <p>c evaluated.</p> | |
| (c)(i) | $a(t) = -9.8$ $v(t) = -9.8t + C$ <p>If $t = 0$ then $v(0) = 2.8$</p> $v(t) = -9.8t + 2.8$ $v(1) = -9.8(1) + 2.8$ $v(1) = -7 \text{ m s}^{-1}$ | <p>Anti-differentiated including constant of integration.</p> | <p>Velocity found.</p> | |

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| <p>(c)(ii)</p> | $v(1) = -9.8(1) + 2.8$ $v(1) = -22.68 \text{ m s}^{-1}$ $-22.68 = -9.8t + 2.8$ $t = \frac{-22.68 - 2.8}{-9.8}$ $t = 2.6 \text{ seconds}$ $h(t) = -\frac{9.8}{2}t^2 + 2.8t + C$ <p>When $t = 2.6$ seconds $h(t) = 0$ (at the water)</p> $0 = -\frac{9.8}{2}(2.6)^2 + 2.8(2.6) + C$ $C = 25.844 \text{ m or } 25.84 \text{ m (2 d.p.)}$ <p>The cliff is 25.84 m above the water.</p> $v(t) = -9.8t + 2.8$ <p>Top of the jump $v(t) = 0$</p> $0 = -9.8t + 2.8$ $t = \frac{2.8}{9.8}$ $t = 0.2857 \text{ seconds (4 d.p.)}$ $t = 0.29 \text{ seconds (2 d.p.)}$ $h(t) = -\frac{9.8}{2}t^2 + 2.8t + 25.84$ $h(0.29) = -\frac{9.8}{2}(0.29)^2 + 2.8(0.29) + 25.84$ $h(0.29) = 26.24 \text{ (2 d.p.)}$ <p>Maximum height above water = 26.24 m</p> | <p>Anti-differentiated to find $h(t)$ with unknown constant of integration</p> <p>OR</p> <p>velocity equation set to 0.</p> | <p>Time of impact found</p> <p>OR</p> <p>time of max height.</p> | <p>T1: Correct answer with some IMS</p> <p>T2: Correct answer with clear correct statements.</p> |
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Evidence Statement

| N0 | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
|------------------------------------|--------------------------|--------|--------|--------|--------|--------|----|----|
| No response; no relevant evidence. | Attempt at one question. | 1 of u | 2 of u | 3 of u | 1 of r | 2 of r | T1 | T2 |

| Q THREE | Expected coverage | Achievement (u) | Merit (r) | Excellence (t) |
|------------|--|--|--|----------------|
| (a) | $f(x) = \frac{6x^3}{3} + \frac{5x^2}{2} - x + c$ $f(1) = 2.5 \text{ implies that } c = -1$ $f(2) = 2(-2)^3 + \frac{5(-2)^2}{2} - (-2) - 1$ $f(2) = 23$ Point is (2,23) | Anti-differentiation correct apart from constant term. | Co-ordinates correct. | |
| (b) | $s(t) = 0.1t^3 + t$ $s(3) = 5.7 \text{ (m)}$ | Correct distance. | | |
| (c) | Red curve. Accept any intercepts with axes.  | Negative cubic shape. OR Positive cubic shape with correct turning points. | Negative cubic shape and turning points correctly located. | |

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| <p>(d)(i)</p> | <p>$A = \text{Area of Triangle} + \text{Area of Rectangle}$</p> $A_T = \frac{(9-y)(2x)}{2} + 2xy$ $A_T = \frac{(9 - (-x^2 + 9))(2x)}{2} + 2x(-x^2 + 9)$ $A_T = -x^3 + 18x$ $A_T' = -3x^2 + 18$ <p>At max, $A_T' = 0$</p> $x^2 = \frac{18}{3}$ $x = \sqrt{6}$ $y = -(\sqrt{6})^2 + 9$ $y = 3$ <p>so the height of the wall is 3</p> <p>($A_T = 12\sqrt{6}$ or 29.39)</p> <p>Possible justifications of the maximum:</p> <ul style="list-style-type: none"> • $A''(\sqrt{6}) = -14.7 < 0$ • $A'(2) = +6, A'(3) = -9$, so slope changes from positive to negative. • $A(2) = 28, A(3) = 27$, both lower than the maximum. • Graph of $A(x)$ is a negative cubic, so the right-hand turning point is the maximum. • Graph of $A'(x)$ is a concave-down parabola, so the right-hand root has gradient going positive to negative. | <p>Their area expression differentiated consistently.</p> | <p>Values for x and / or y found.</p> | <p>Height of wall clearly stated AND Justification from: Graph of function, or gradient function, gradient on each side of the points, second derivative or substitution in to the function.</p> |
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| (d)(ii) | <p>$A = \text{Area of Triangle} + \text{Area of Rectangle}$</p> $A_T = \frac{(d-y)(2x)}{2} + 2xy$ $A_T = \frac{(d - (-kx^2 + d))(2x)}{2} + 2x(-kx^2 + d)$ $A_T = -kx^3 + 2dx$ $A_T' = -3kx^2 + 2d$ <p>At max, $A_T' = 0$</p> $x^2 = \frac{2d}{3k}$ $x = \sqrt{\frac{2d}{3k}}$ $y = -k\left(\sqrt{\frac{2d}{3k}}\right)^2 + d$ $y = \frac{-2d}{3} + d$ $y = \frac{d}{3}$ $A_{\text{Triangle}} = \frac{(d-y)(2x)}{2}$ $A_{\text{Triangle}} = \frac{\left(d - \left(\frac{d}{3}\right)\right)\left(2\sqrt{\frac{2d}{3k}}\right)}{2}$ $A_{\text{Triangle}} = \frac{2d}{3} \sqrt{\frac{2d}{3k}}$ $A_{\text{Rectangle}} = 2xy$ $A_{\text{Rectangle}} = 2\left(\sqrt{\frac{2d}{3k}}\right)\left(\frac{d}{3}\right)$ $A_{\text{Rectangle}} = \frac{2d}{3} \sqrt{\frac{2d}{3k}}$ $A_{\text{Rectangle}} = A_{\text{Triangle}}$ | Relationship formed and correctly differentiated. | Values for x or y found. | Area for triangle and rectangle found separately and shown to be equal. |
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| N0 | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
|------------------------------------|--------------------------|--------|--------|--------|--------|--------|--------|--------|
| No response; no relevant evidence. | Attempt at one question. | 1 of u | 2 of u | 3 of u | 1 of r | 2 of r | 1 of t | 2 of t |

Cut Scores

| Not Achieved | Achievement | Achievement with Merit | Achievement with Excellence |
|--------------|-------------|------------------------|-----------------------------|
| 0 – 6 | 7 – 13 | 14 – 19 | 20 – 24 |