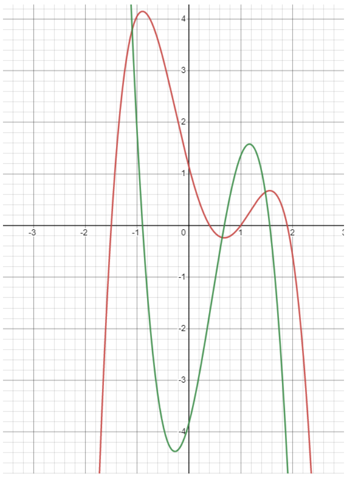
Assessment Schedule – 2023

Mathematics and Statistics: Apply calculus methods in solving problems (91262)

Evidence

Q	Evidence	Achievement	Merit	Excellence
ONE (a)	$f'(x)=8x-12$ $4=8x-12$ $x=2$ $f(2)=4(2)^2-12(2)$ $f(2)=-8$ The point on the curve $f(x)$ that has a gradient equal to 4 is (2,–8).	• Derivative found AND x-coord found.		
(b)	$f'(x)=4x^3-6x^2-4x$ $f(x)=\frac{4x^4}{4}-\frac{6x^3}{3}-\frac{4x^2}{2}+C$ $f(x)=x^4-2x^3-2x^2+C$ at $(2,-5)$ $-5=(2)^4-2(2)^3-2(2)^2+C$ $C=3$ $f(x)=x^4-2x^3-2x^2+3$	• Correct anti-derivative with $+C$.	• Correct $f(x)$ including $C=3$.	
(c)	$f'(x)=\frac{4x^3}{4}-\frac{6x^2}{3}-24x$ $f'(x)=x^3-2x^2-24x$ $f'(x)=x(x-6)(x+4)$ Turning points when x = 0, 6, –4 $f''(x)=3x^2-4x-24$ $f''(0)=3(0)^2-4(0)-24$ $f''(0)=-24$ (negative, $\therefore$ local max) $f''(6)=60$ (positive, $\therefore$ local min) $f''(-4)=40$ (positive, $\therefore$ local min) Function is increasing $-4<x<0$ and $6<x$.	• Derivative found. AND Set = 0 or implied.	• x-coordinates found.	• Justification for regions where the function is increasing. Justification includes: f'', checking gradients or graph of function or similar.
(d)	$A=x\times 2r$ $p=2x+2\pi r$ $400=2x+2\pi r$ $x=200-\pi r$ $A=(200-\pi r)\times 2r$ $A=400r-2\pi r^2$ $A'=400-4\pi r$ $0=400-4\pi r$ $r=\frac{100}{\pi}=31.83$ (2 d.p.) $x=200-\pi\left(\frac{100}{\pi}\right)$ $x=100$ m $A''=-4\pi$ (negative, therefore local max) (Maximum area is 6366.20 m^2)	• Relationship formed AND differentiated.	• Differentiated AND values of r and x found.	• Justification of max confirmed.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	One point made incompletely.	1u	2u	3u	1r	2r	1t	2t

Q	Evidence	Achievement	Merit	Excellence
TWO (a)	 Green line is $f'(x)$ (answer to this question). Red line is the original function $f(x)$.	• TWO of 3 required: - shape - orientation - x-intercepts.	• ALL three required: - shape - orientation - x-intercepts.	
(b)	$f'(x) = 9x^2 - 1$ Gradient of line $y = 8x + 10$ is 8. $8 = 9x^2 - 1$ $x = \pm 1$ $f(1) = 6$ $f(-1) = 2$ Substituting in (1,6) $y - 6 = 8(x - 1)$ $y = 8x - 2$ wrong Substituting in (–1,2) $y - 2 = 8(x - 1)$ $y = 8x + 10$ correct	• Derivative found AND set to 8.	• Uses (–1,2) to form tangent equation $y = 8x + 10$.	

(c)(i) (ii)	$a = -2.5\text{ m s}^{-2}$ $v = -2.5t + C_1$ At $t = 0, v = 27.78$ $v = -2.5t + 27.78$ For $v = 0$ $0 = -2.5t + 27.78$ $t = 11.11\,s$ $s = -\frac{2.5t^2}{2} + 27.78t + C_2$ At $t = 0, s = 0$ $s = -\frac{2.5t^2}{2} + 27.78t$ For $v = 0, t = 11.11$ $s = -\frac{2.5(11.11)^2}{2} + 27.78(11.11)$ $s = 154.35\text{ m}$ The car travels 154.35 m between the start of the deceleration to the point where the car is completely stopped	• General expression for v found.	• Finds general expression for s and solves to find the time it takes the car to reach a complete stop. t = 11.112 s is found in (c)(i).	Finds value of C_2 AND correctly shows distance car travelled before coming to complete stop.

(iii)	$a = 2.1 \text{ m s}^{-2}$ $v = -2.1t + C$ At $t = 0, v = k$ $v = -2.5t + k$ For $v = 0$ $0 = -2.1t + k$ to stop $t = \frac{k}{2.1}$ or $k = 2.1t$ $s = -\frac{-2.1t^2}{2} + kt + C_2$ At $k = 22.1t, v = 0. C_2 = 0$ $s = -105t^2 + 2.1t^2$ So $t = \pm 12.1$ $k = 2.1 \times 12.1$ $k = 25.43 \text{ m s}^{-1}$ **Alternatively** $a = 2.1 \text{ ms}^{-1}$ $v = -2.1t + C_1$ At $t = 0, v = k$ $v = -2.1t + k$ $t = \frac{k}{2.1}$ $s = -\frac{2.1t^2}{2} + kt + C_2$ At $t = 0, s = 0 \therefore C_2 = 0$ $s = -\frac{2.1t^2}{2} + kt$ For $v = 0, t = \frac{k}{2.1}$ $s = 154.35 \text{ m}$ $154.35 = -\frac{2.1\left(\frac{k}{2.1}\right)^2}{2} + k\left(\frac{k}{2.1}\right)$ $k = 25.5 \text{ m s}^{-1}$ The car with older brakes needs to be travelling at an initial velocity of 91.55 km/hr in order to stop in the same distance.	• General expression for found.	• Expression for t found. AND General expression for s.	• Value for k (unknown initial velocity) found.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	One point made incompletely.	1u	2u	3u	1r	2r	1t	2t

Q	Evidence	Achievement	Merit	Excellence
THREE (a)(i)	$y' = 6x^2 - 18x + 12$ $y'(1) = 6(1)^2 - 18(1) + 12$ $y' = 0$	• Derivative found AND correct gradient.		
(ii)	$y' = 6(x-1)(x-2)$ $0 = 6(x-1)(x-2)$ $x = 1$ or $x = 2$ $y(2) - 7$ Other coordinate with gradient = 0 is (2,7)		• Correct gradient equation equated to 0 AND coordinates found.	
(b)(i)	$F' = 15t^2 - 1680t + 42\,180$ $F'(5) = 15(5)^2 - 1680(5) + 42\,180$ $F'(5) = 34\,155$	• Derivative found AND correct rate found.		
(ii)	$F' = 15t^2 - 1680t + 42180$ $0 = t^2 - 112t + 2812$ $t = 74$ or 38 $F(38) = 664\ 240$ (local max) $F(74) = 547\ 600$ (local min)	• Derivative set to 0 or implied AND solved.	• Maximum found.	
(iii)	Turning points at t = 38 days and t = 74 days. Positive cubic function – increasing / decreasing / increasing. Only losing followers between 38 and 74 days 74–38 = 36 days $F'' = 1530t - 1680$ $F''(38) = -540 \rightarrow$ negative $\therefore$ local maximum $F''(74) = 540 \rightarrow$ positive $\therefore$ local minimum			• Days found, justifying the decreasing region.

(c)	$f'(x)=3x^2-2px+21$ $f'(4)=69-8p$ $f(x)=x^3-px^2+21x-7$ $f(4)=(4)^3-p(4)^2+21(4)-7$ $f(4)=141-16p$ Equation of the tangent: $y=mx-7$ $y=(69-8p)x-7$ $141-16p=(69-8p)\times 4-7$ $141-16p=269-32p$ $p=8$ **Alternatively** $f'(x)=3x^2-2px+21$ $f'(4)=69-8p$ $f(4)=(4)^3-p(4)^2+21(4)-7$ $f(4)=141-16p$ Two points $(4,141-16p)$ and $(0,-7)$ $m=\frac{\left((141-16p)--7\right)}{4-0}$ $m=\frac{148-16p}{4}$ $f'(4)$ must equal m $\frac{148-16p}{4}=69-8p$ $148-16p=276-32p$ $p=8$	• Derivative found and expression for the gradient of tangent found in terms of p.	• Formed tangent equation. OR Both expressions for the gradient of tangent found in terms of p.	• Value for p found.

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No response; no relevant evidence.	One point made incompletely.	1u	2u	3u	1r	2r	1t	2t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 13	14 – 19	20 – 24