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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

2

SUPERVISOR'S USE ONLY

Level 2 Mathematics and Statistics, 2014

91262 Apply calculus methods in solving problems

2.00 pm Wednesday 19 November 2014
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–19 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

(a) A function g is given by $g(x) = x^3 - 4x + 5$.

Find the gradient of the graph of g at the point where $x = 2$.

(b) For a function f

$f'(x) = 3x^2 + 4x - 1$

The graph of f passes through the point (2,5).

Find the equation of the function f.

ASSESSOR'S USE ONLY

(c) A diver dives into a pool. The depth d metres that she reaches t seconds after she hits the water is given by

$$d(t) = 1.25t^2 - 4t$$

Find the greatest depth that she reaches.

(d) The graph of $f(x) = -x^2 + kx - 5$ has a turning point at (3,4).

Find the gradient of the function at the point where $x = 4$.

(e) $g(x) = \frac{2x^3}{3} + \frac{3x^2}{2} - 20x + 4$

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For what values of x is g a decreasing function?

You must use calculus in finding your solution.

(f) During a fund-raising cycle ride, the distance s kilometres of a cyclist from a fixed point on his ride is modelled by the function

$$s(t) = 0.1875t^3 - 2.25t^2 + 21t + 0.5$$

where t is the time in hours since the cyclist passed the fixed point.

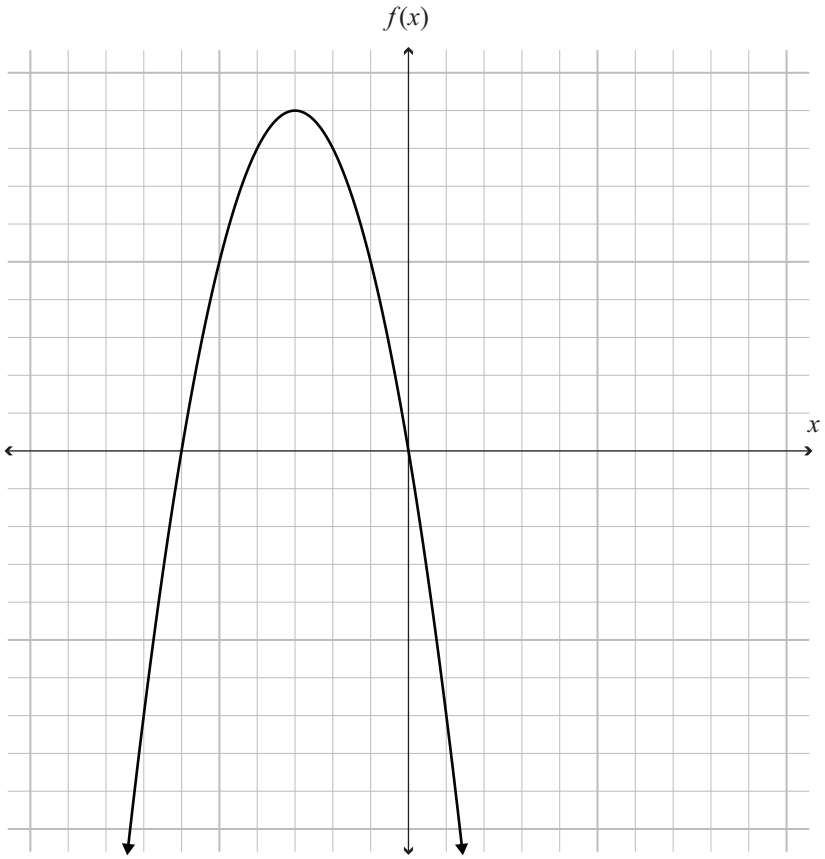
How far will the cyclist be from the fixed point when he reaches his minimum speed?

ASSESSOR'S USE ONLY

QUESTION TWO

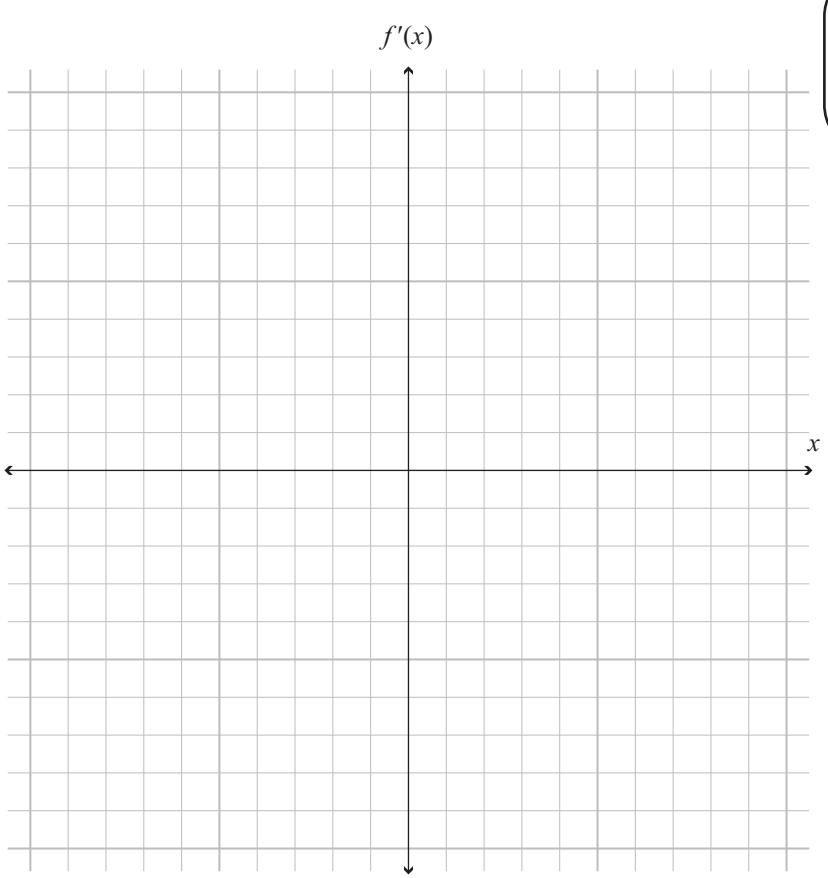
ASSESSOR'S USE ONLY

(a) The graph of the function $y = f(x)$ is shown on the axes below.



On the axes below sketch the gradient function $y = f'(x)$.

If you need to redraw this graph, use the grid on page 16



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(b) The graph of a function $y = g(x)$ passes through (0,0), and its gradient function is $g'(x) = 2x - 5$.

Find the y-coordinate of the point on the curve where $x = 3$.

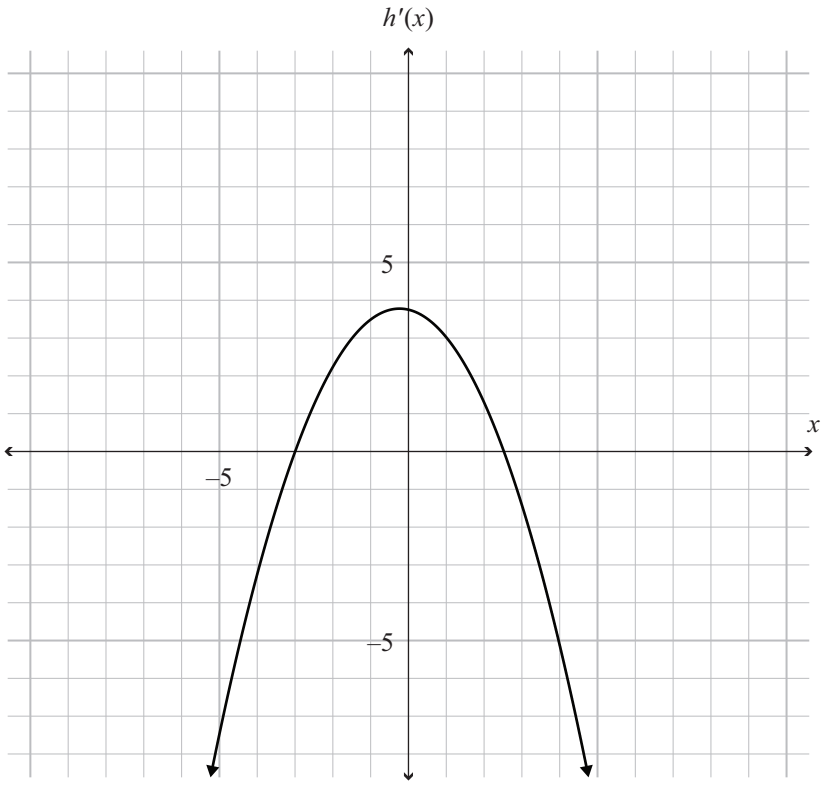
(c) Sione claims that the resale value of a car, t years after it is sold, can be modelled by the function

$R = 150t^2 - 2250t + 38\,000$, where R is the resale value in dollars.

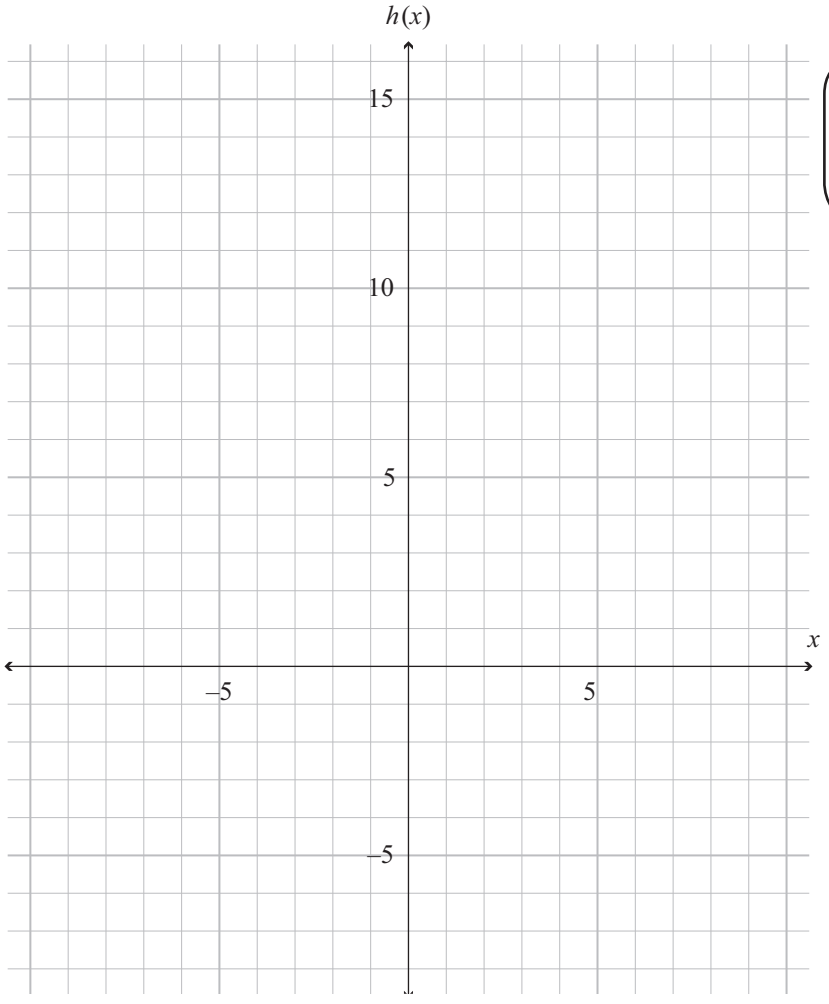
How long after it is sold will the car's value be changing at a rate of –$150 per year?

ASSESSOR'S USE ONLY

(d) A gradient function $h'(x)$ is shown on the axes below.



Sketch the function $h(x)$ if $h(x)$ passes through the point $(-3,2)$.



If you need to redraw this graph, use the grid on page 17

(e) The gradient of the graph of a function is given by $\frac{dy}{dx} = 9x - 3x^2$.

ASSESSOR'S USE ONLY

At the maximum turning point of the graph of the function, $y = -4$.

Find the equation of the graph.

Note: The turning point where $y = -4$ must be shown to be a maximum.

(f) The tangent to the graph of the function $y = px^2 + 4x - 5$ at the point where $x = 3$ passes through the point (0,4).

Find the value of p, and hence the equation of the tangent.

ASSESSOR'S USE ONLY

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The examination continues on the following page.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) Find the x-coordinate of the point on the graph of the function $f(x) = 4x - x^2$ where the gradient is equal to 10.

(b) A swimming pool is being filled.

At the deepest point, the depth of the water in the pool at any instant is h m.

The volume of water in the pool V m^3 is given by

$$V(h) = 20h^2 + 40h$$

Find the rate at which the volume of water is changing with respect to the depth when the water in the pool is 0.75 m deep.

(c) A balloon is being inflated with helium.

The volume V cm^3 of helium in the balloon is given by $V = \frac{4}{3}\pi r^3$, where r cm is the radius of the balloon.

Find the rate at which the volume of gas in the balloon is changing with respect to the radius when the volume is 288π cm^3.

ASSESSOR'S USE ONLY

(d) The length of a cuboid is three times its width.

The sum of the height, width, and length is 150 cm.

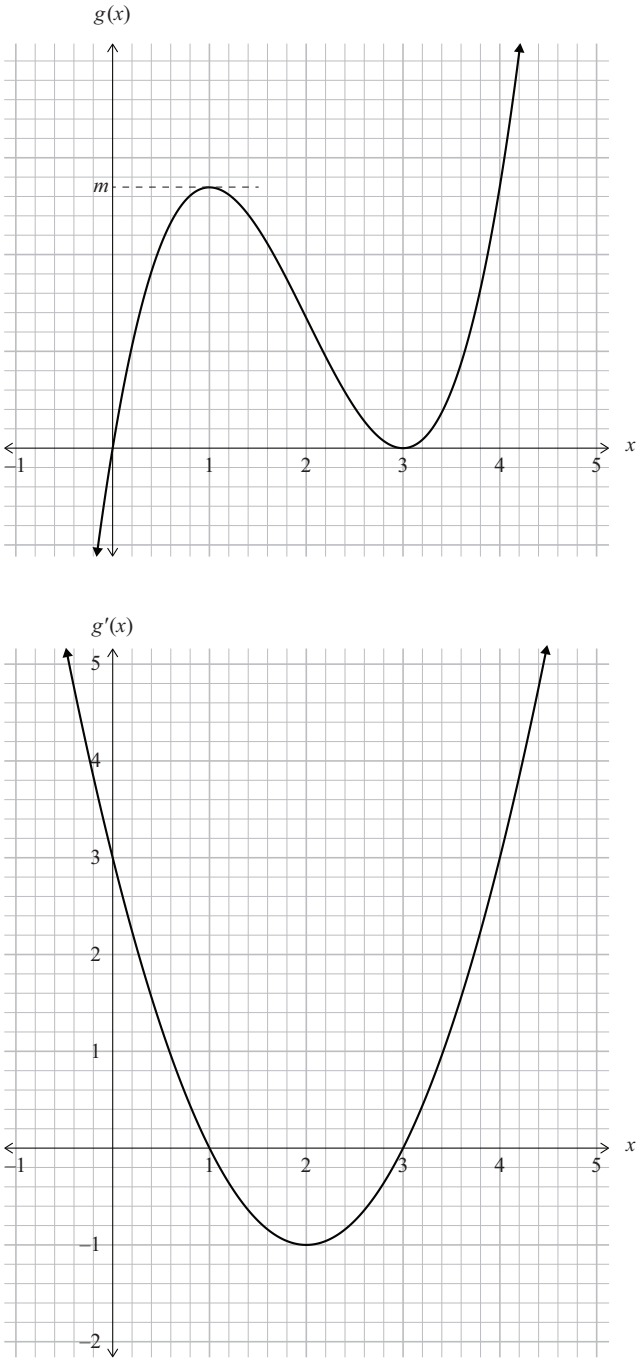
The volume V cm^3 can be expressed as

$V = 450x^2 - 12x^3$ where the width is x cm.

Find the height of the cuboid for which the volume is maximum.

ASSESSOR'S USE ONLY

(e) The graph of a function $g(x)$ and the graph of its gradient function, are shown below.



Find the value m, the y-value for the maximum turning point of the function $g(x)$.

ASSESSOR'S USE ONLY

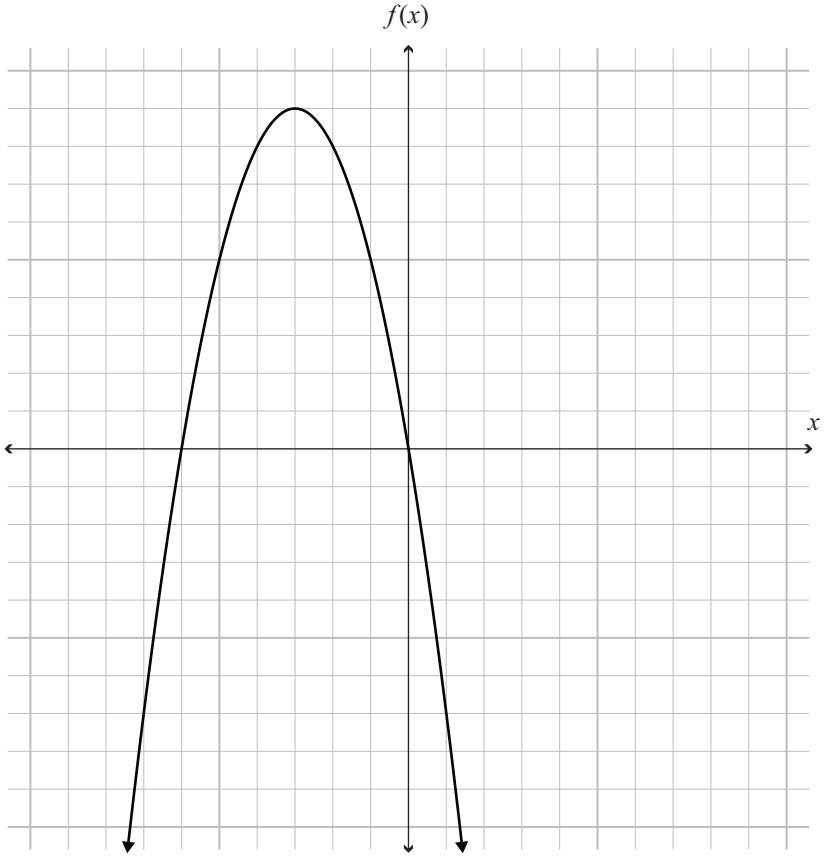
(f) An aircraft is travelling at 70 m s^{-1} when it lands.

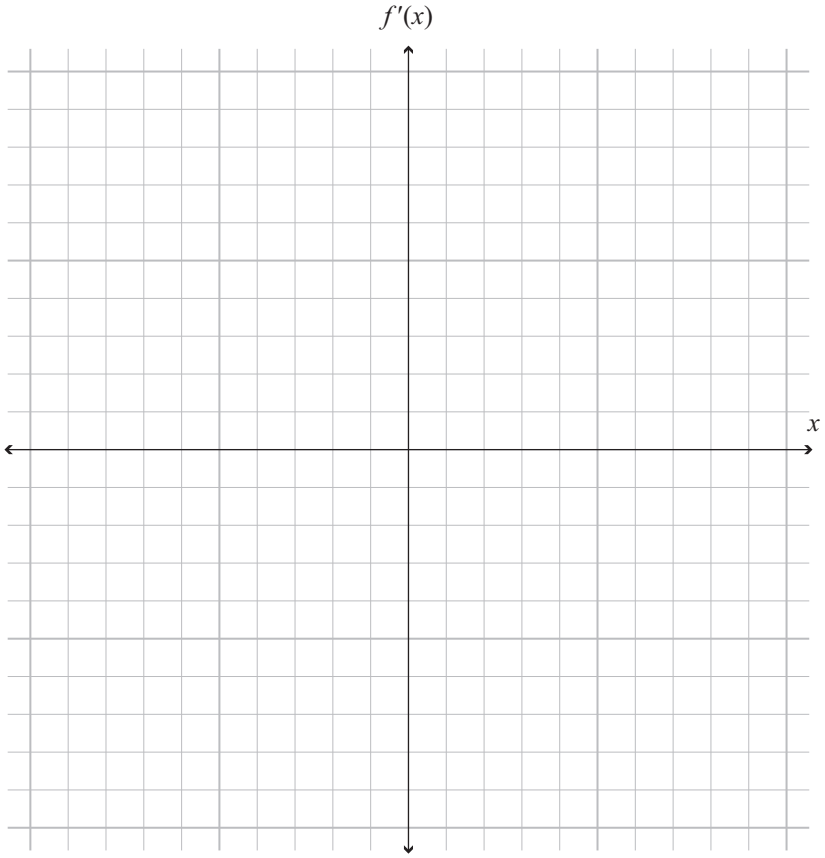
Its speed changes at a constant rate of –3.3 m s^{-2}.

Use calculus to find how far the aircraft will travel from where it lands to where it has a speed of 4 m s^{-1}.

If you need to redraw your graph from Question Two (a), draw it on the lower grid. Make sure it is clear which answer you want marked.

ASSESSOR'S USE ONLY





If you need to redraw your graph from Question Two (d), draw it on the lower grid. Make sure it is clear which answer you want marked.

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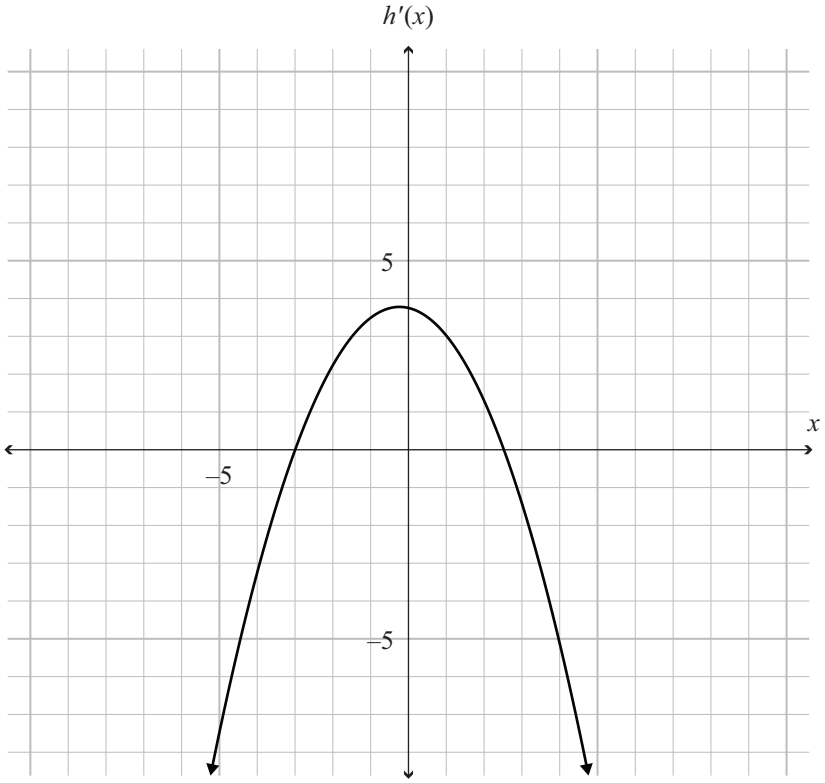

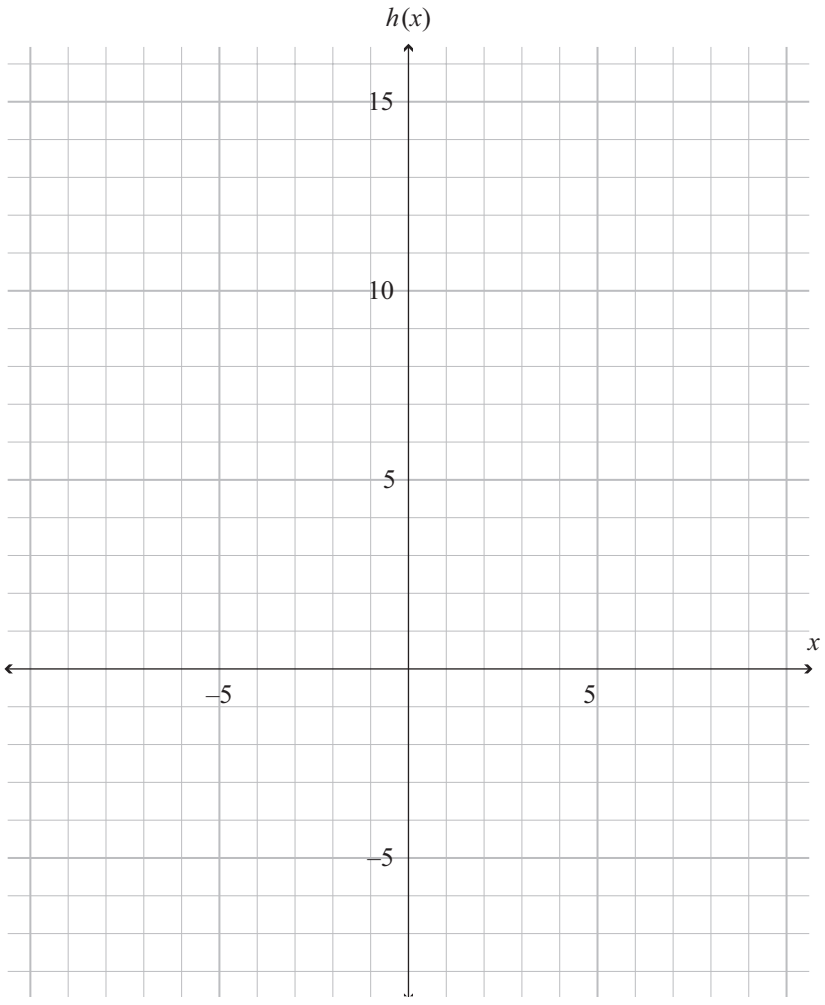

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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