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QUALIFY FOR THE FUTURE WORLD
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Level 2 Mathematics and Statistics, 2019

91262 Apply calculus methods in solving problems

9.30 a.m. Thursday 21 November 2019
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

You must show the use of calculus in answering all questions in this paper.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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(a) A function f is given by $f(x) = x^4 + 3x^2 - 17$

Use calculus to find the gradient of the graph of the function at the point where $x = 2$.

(b) Find the coordinates of the point(s) on the graph of the function $y = 4x^3 - 4x + 4$ where the tangent to the curve is parallel to the line $y - 8x + 6 = 0$.

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(c) Sophie is blowing up a balloon. The volume of the balloon is given by $V = \frac{4}{3}\pi r^3$

where V is the volume of the balloon in cm^3, and r is the radius in cm.

Find the radius of the balloon when the rate of change of the volume with respect to the radius is 25π cm^3 / cm.

(d) Use calculus to find the quadratic function of x that has the following properties:

- a rate of increase (gradient) of 7 when $x = 0$
- a turning point when $x = 1$
- a value of -20 when $x = 4$.

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(e) The graphs of

$g(x) = x^3 - ax^2 + 6$ and $h(x) = 2x^2 + bx + 13$

just touch when $x = -1$ (so they have a common tangent at the point of contact).

Use calculus to find the coordinates of the point of contact of the two graphs.

QUESTION TWO

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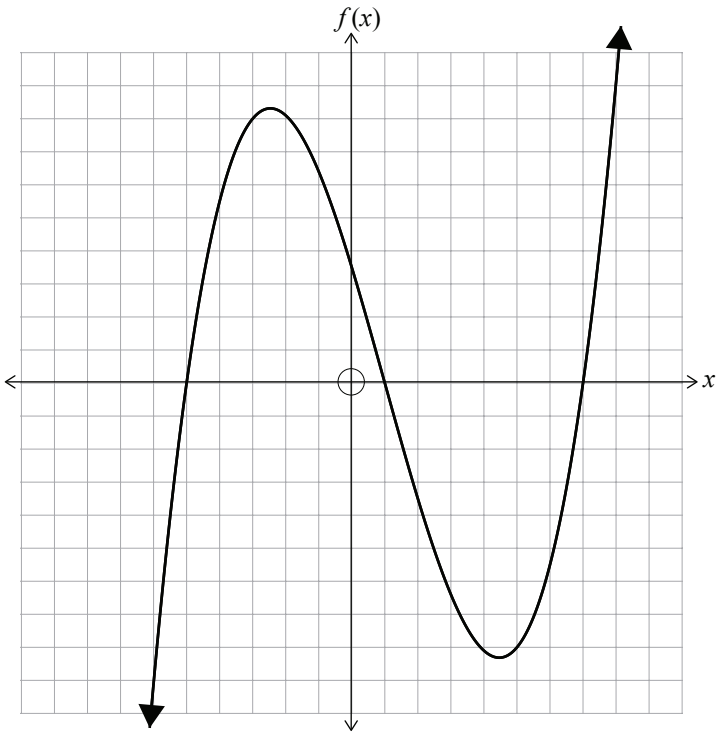
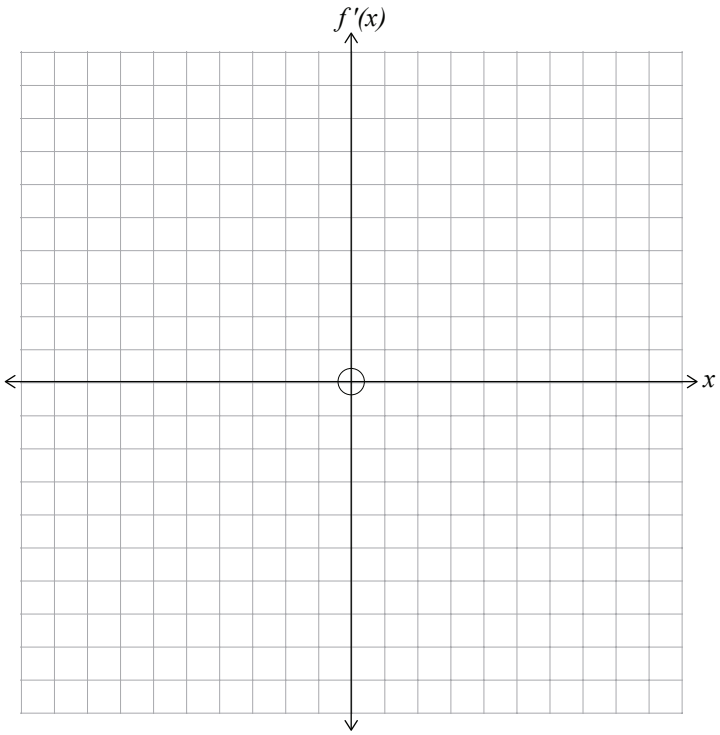
(a) The graph of a function $y = f(x)$ is shown on the axes below.

Both sets of axes have the same scale.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.



If you need to redo this question part, use the grids on page 12.

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(b) A function, $p(x)$, has a derived function given by $p'(x) = 5 - 8x^3$.

The graph of $p(x)$ passes through $(2,-25)$.

Find the equation of $p(x)$.

(c) Consider the graph of the function $f(x) = -2k^2x^3 + 3kx^2 + 12x - 55$, where k is a positive constant.

Use calculus to find expressions, in terms of k, for the range(s) of values of x for which this graph is increasing.

Justify your choice of range(s) clearly.

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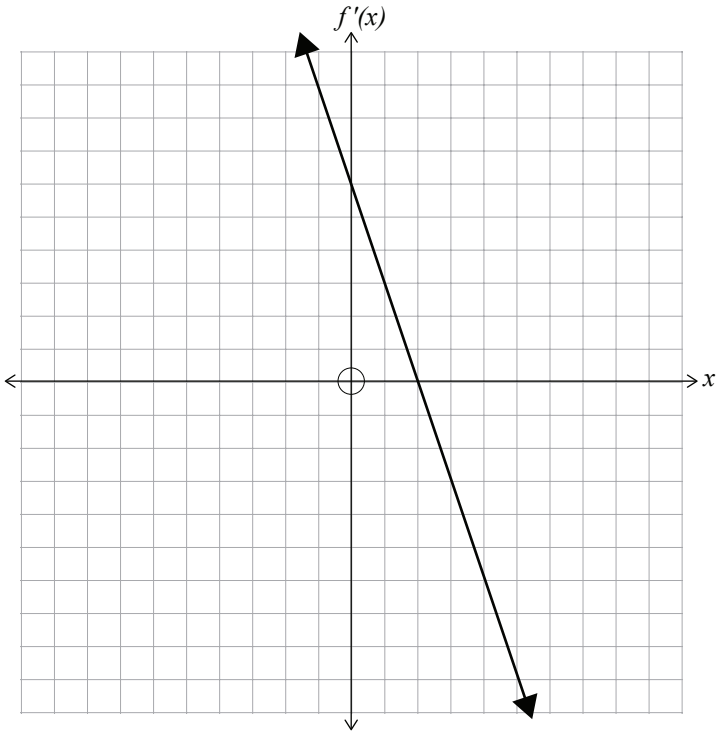
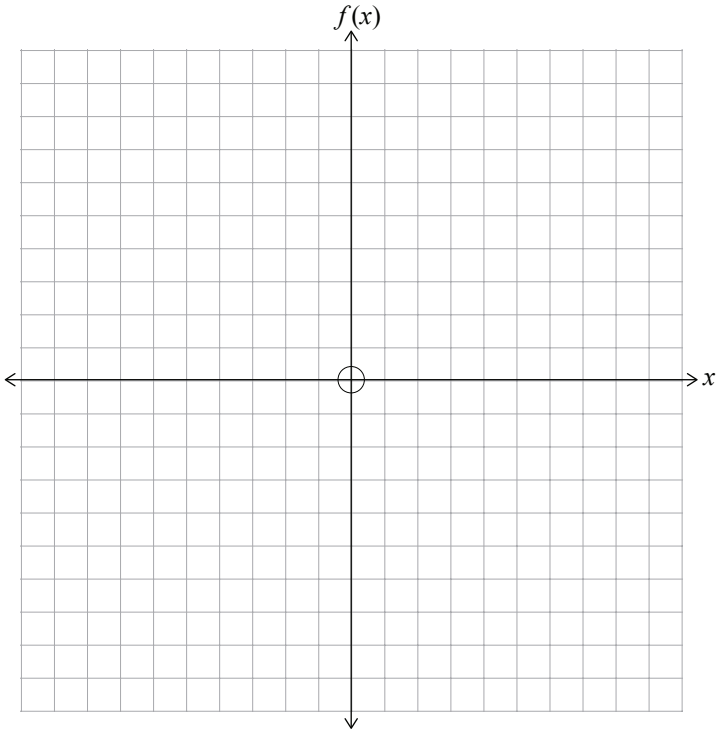
(d) The diagram below shows the graph of the gradient function $y = f'(x)$ of a function $y = f(x)$. Both sets of axes have the same scale.



The graph of the function passes through the origin (0,0).

On the axes below, sketch the graph of the function $f(x)$.



If you need to redo this question part, use the grids on page 13.

(e) A car travelling at a constant speed of 28 m s^{-1} on a straight road is approaching a corner. The driver applies the brakes and decelerates at a constant rate of 4 m s^{-2} until the car reaches the corner with a speed of 10 m s^{-1}.

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Use calculus to find how far the car was from the corner when the driver first applied the brakes.

Justify your answer.

QUESTION THREE

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(a) Matiu is operating his remote-controlled car, which is moving back and forth along a straight track. The car's distance s metres from a point P on the track, t seconds after leaving P, is given by $s(t) = 6t - t^2$.

(i) Find the initial speed of the car.

(ii) How long after leaving P does the car change direction?

(iii) How fast is the car moving when it reaches P for the second time?

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(b) A parcel in the shape of a rectangular cuboid with a square cross section is to be sent through the post. The sum of the length of the cuboid and the perimeter of the square cross section is to be 100 cm.

Diagram is NOT to scale

Find the maximum possible volume of the parcel.

Explain how you know that your answer is the maximum, not the minimum, volume.

(c) There are two points, A and B, on the graph of the function $f(x) = x^3 - 3x^2 - 4x$ where the tangent to the graph passes through the origin.

Find the coordinates of points A and B and the equation of each tangent.

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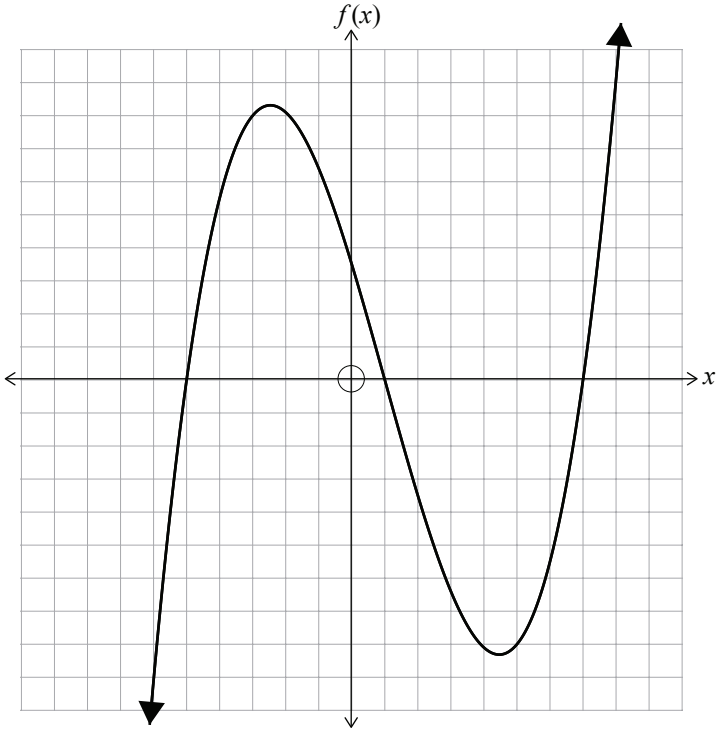
SPARE GRIDS

If you need to redo Question Two (a), use the grid below. Make sure it is clear which answer you want marked.

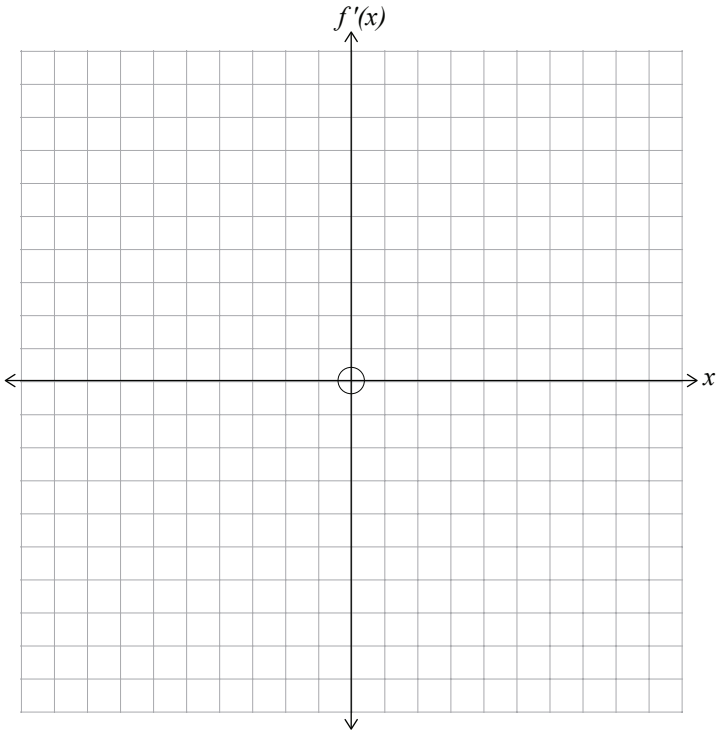
QUESTION TWO

(a) The graph of a function $y = f(x)$ is shown on the axes below.

Both sets of axes have the same scale.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.



If you need to redo Question Two (d), use the grid below. Make sure it is clear which answer you want marked.

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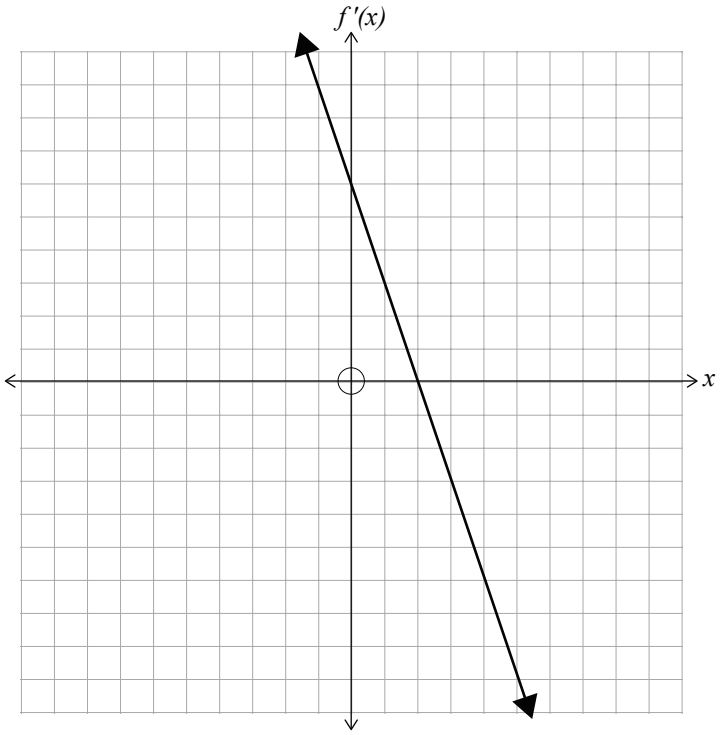
QUESTION TWO

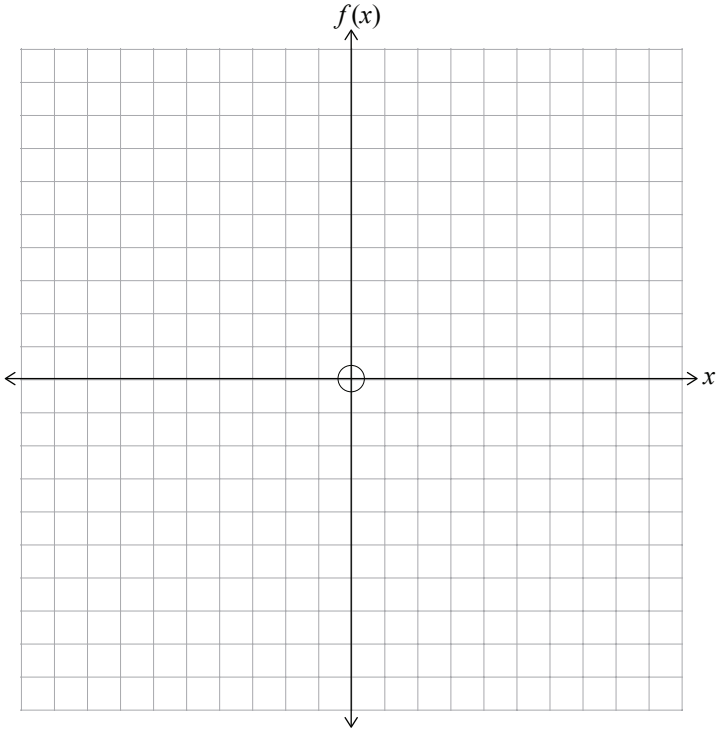
(d) The diagram below shows the graph of the gradient function $y = f'(x)$ of a function $y = f(x)$.

Both sets of axes have the same scale.



The graph of the function passes through the origin (0,0).

On the axes below, sketch the graph of the function $f(x)$.



Extra space if required.
Write the question number(s) if applicable.

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