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Tick this box if there is no writing in this booklet

Level 2 Mathematics and Statistics 2020

91262 Apply calculus methods in solving problems

9.30 a.m. Thursday 19 November 2020
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

You must show the use of calculus in answering all questions in this paper.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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(a) A function is given by $f(x) = x^3 - 2x^2 + 5$.

Find the gradient of the graph at the point where $x = 4$.

(b) Another function is given by $h(x) = 0.5x^2 + 3x - 1$.

Find the x-coordinate of the point on the graph of this function where the gradient is 5.

(c) Find the equation of the tangent to the curve of $y = x^2 + 5x$ at the point (2,14).

(d) The gradient function of a curve is given by $f'(x) = px - 4$, where p is a constant.
The curve passes through the points (4,12) and (–6,2).

Find the equation of the curve.

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(e) A fishing boat is 80 km from its port when it reaches its fishing grounds.

Having reached its fishing grounds, the boat accelerates in a straight line directly away from its port as it catches fish. The acceleration of the boat is given by $a(t) = 0.5$ km h^{-2}, where t is the number of hours since it started fishing.

The speed of the boat is 3 km h^{-1} when it starts fishing.

During which hour did the boat travel 11.75 km? You must use calculus to find your answer.

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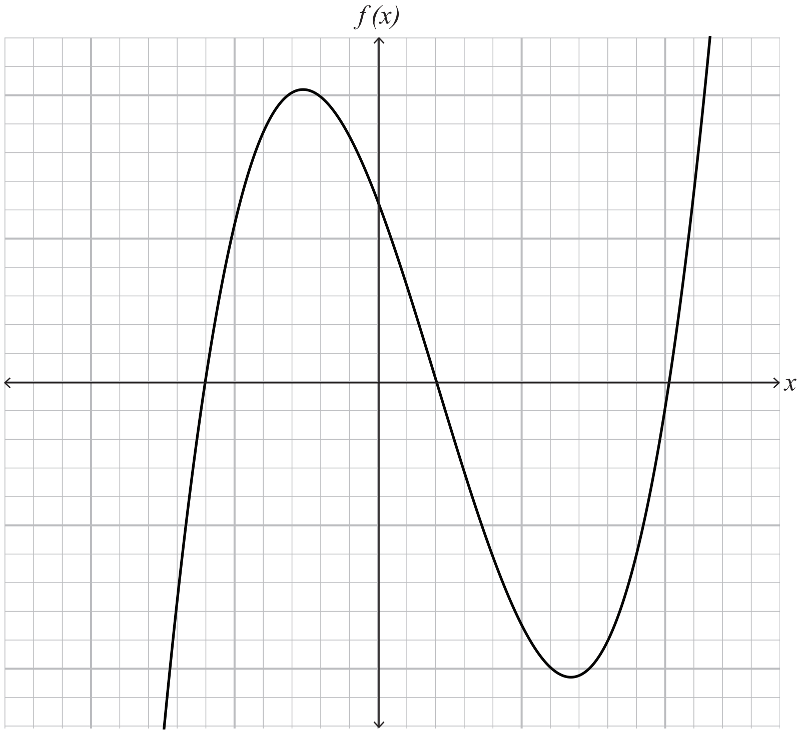
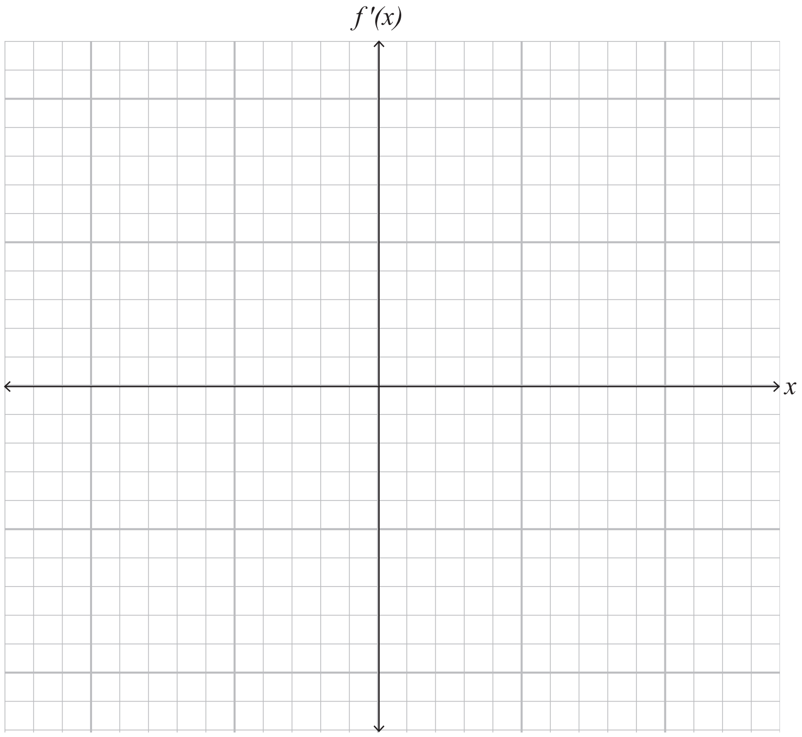
QUESTION TWO

(a) The graph of a function $y = f(x)$ is shown on the axes below.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.

Both sets of axes have the same horizontal scale.



If you need to redraw this graph, use the grid on page 14.

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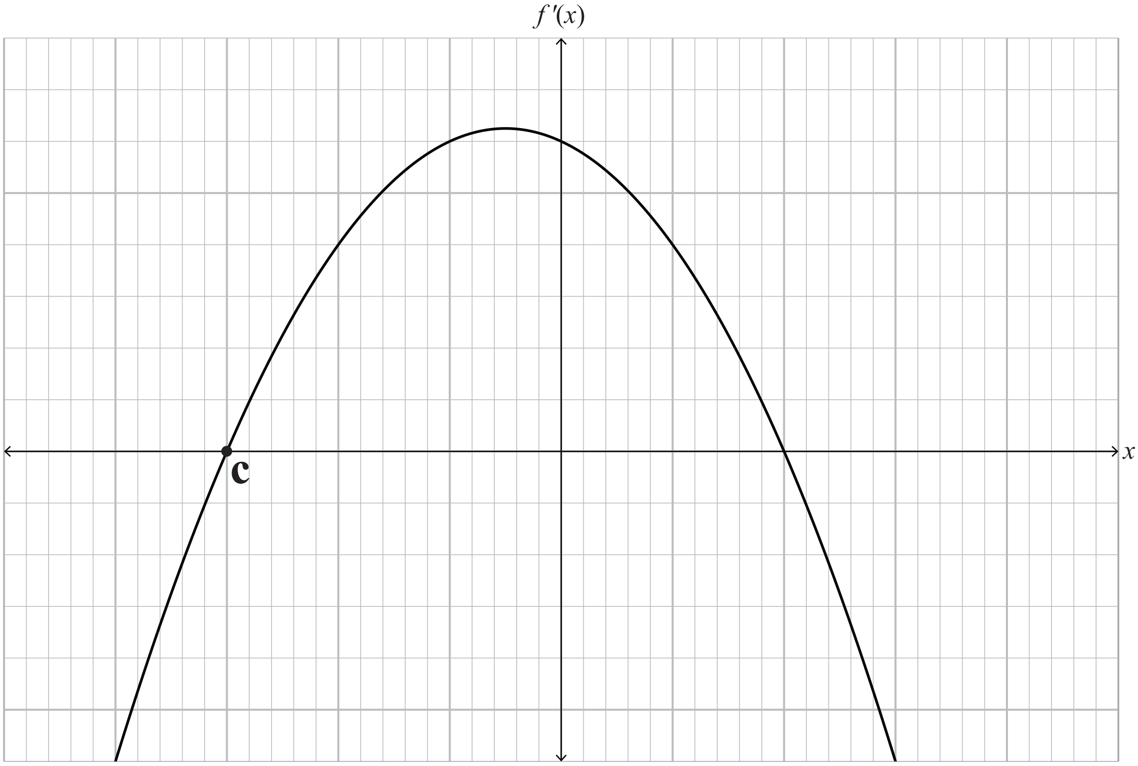
(b) Find the x-coordinate of the maximum point on the curve given by $y = 2x^3 - 42x^2 + 240x + 8$. Explain how you know that this point is the maximum, not the minimum, point.

(c) The speed of an object is given by $v(t) = 3t^2 - 5t$ m s^{-1}, where t is measured in seconds.

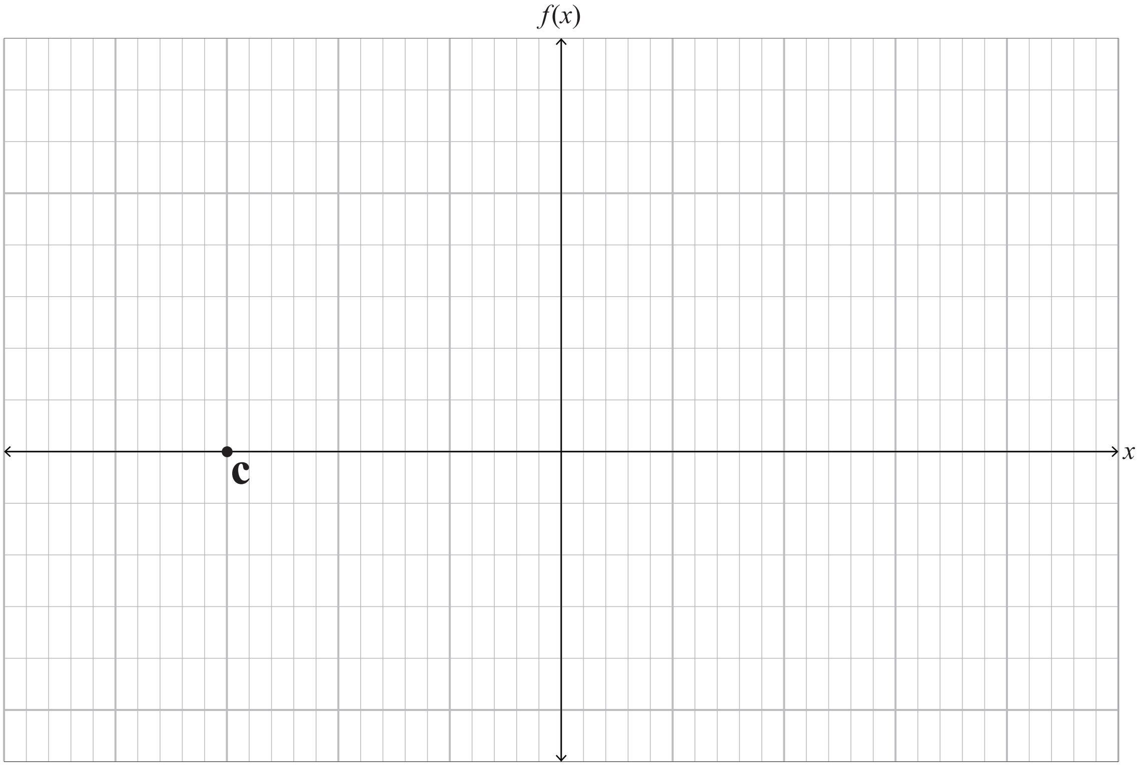
What is the object's acceleration when t = 2?

(d) The graph of a gradient function, $f'(x)$, is shown on the axes below. The point (c,0) is on this graph.



Sketch the graph of the function $f(x)$ on the axes below, given that $f(\text{c}) = 0$.

Both sets of axes have the same horizontal scale.



If you need to redraw this graph, use the grid on page 15.

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(e) A cubic function, $f(x) = ax^3 + bx^2 + cx + d$, has turning points that occur at $x = 3$ and $x = –5$.

Find an expression for c in terms of a.

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QUESTION THREE

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(a) $f'(x) = 3x^2 - 2x - 4$ is the derivative of a function f.

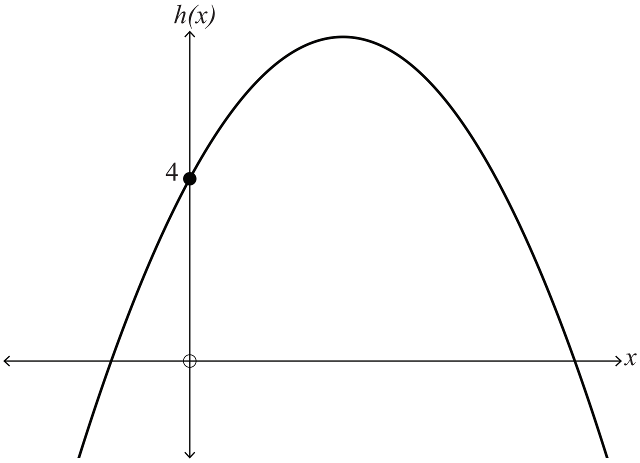
The graph of $f(x)$ passes through the point (3,10).

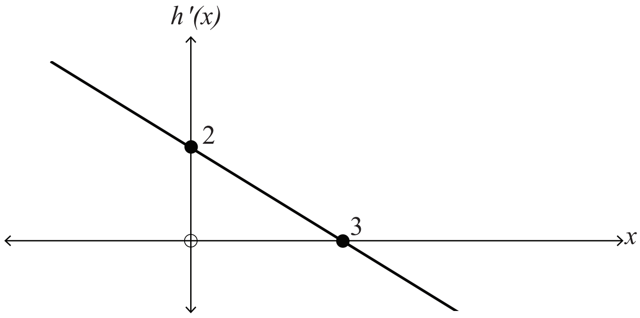
Find an expression for $f(x)$.

(b) A whale surfaces on a still sea. As the whale surfaces, a circular ripple expands outwards at a constant speed, so that the radius of the circle is given by $r = 0.7t$ metres t seconds after the whale surfaces.

At what rate is the area of water within the circular ripple increasing, 20 seconds after the whale surfaces?

(c) The graph of the quadratic function $h(x)$ together with that of its gradient function $h'(x)$ are given below.





Find an expression for $h(x)$.

You must use calculus to obtain your answer.

(d) The function $f(x) = kx^3 + 9x$ has a tangent with a gradient of 15 where $x = 2$.

Find the value of k.

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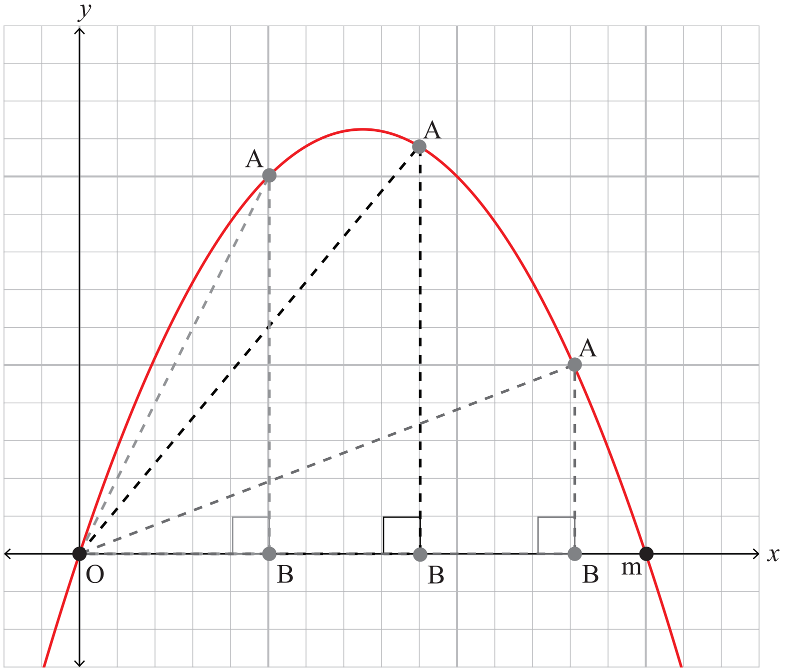
Please turn over ➤

(e) A right-angled triangle OAB is drawn within a parabola, as shown in the diagram below. Three possible triangles OAB have been drawn.

The point O is the origin (0,0), and point A can lie anywhere on this parabola above the x-axis.

The equation of the parabola is of the form $y = mx - x^2$, where m is a positive constant.



Use calculus to find an expression, in terms of m, for the maximum possible area of triangle OAB.

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SPARE GRIDS

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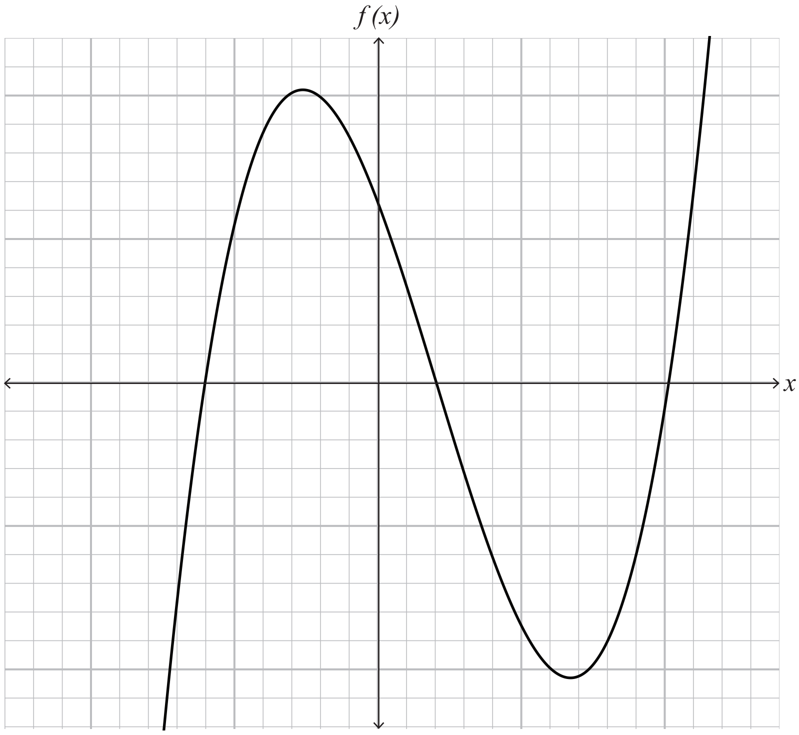
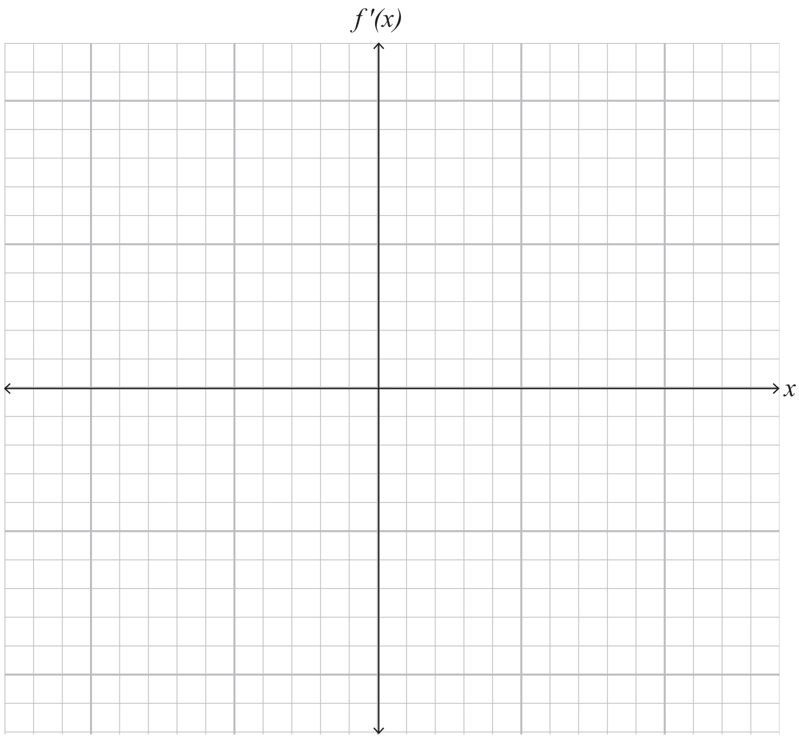
If you need to redo Question Two (a), use the grid below. Make sure it is clear which answer you want marked.

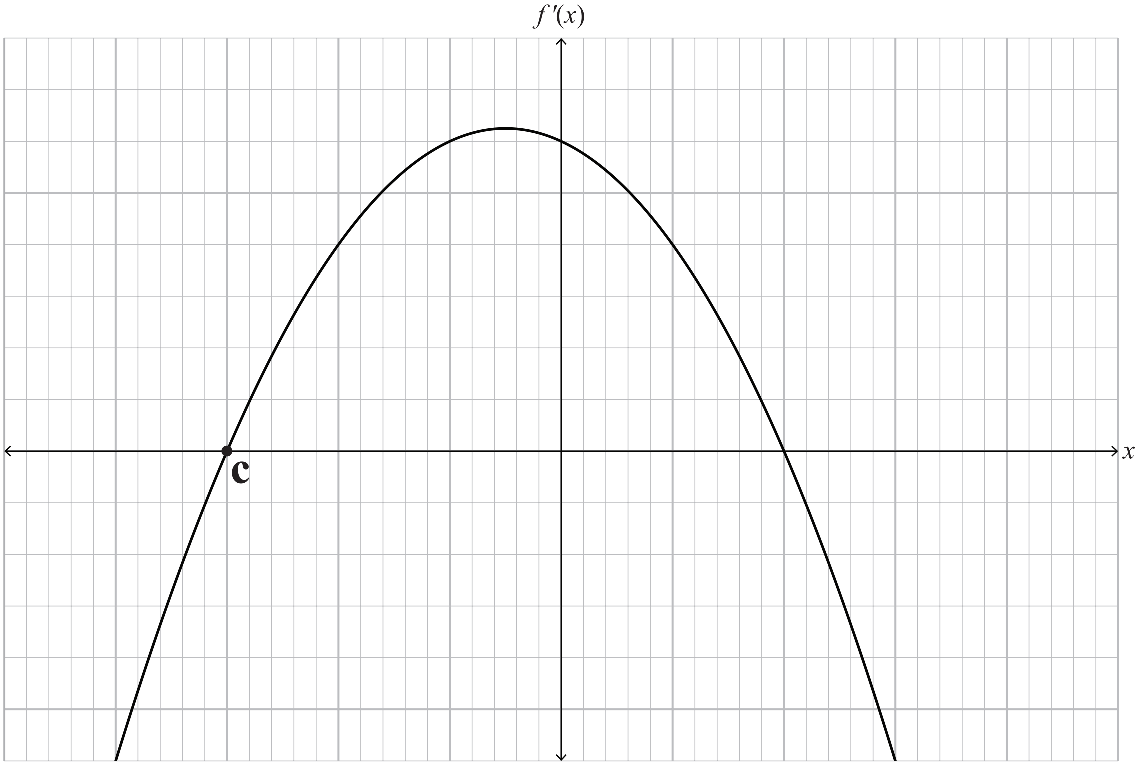
(a)

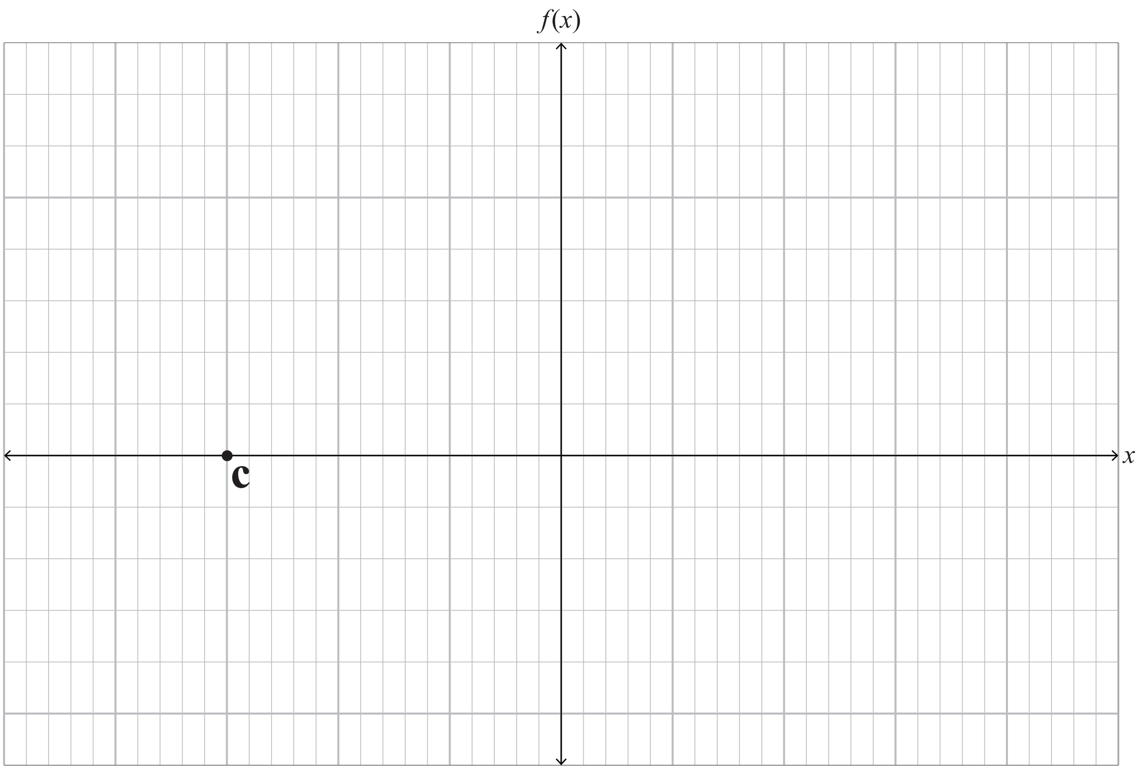




If you need to redo Question Two (d), use the grid below. Make sure it is clear which answer you want marked.

(d)





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Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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