91262

912620

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

2

SUPERVISOR'S USE ONLY

Tick this box if you have NOT written in this booklet

Level 2 Mathematics and Statistics 2021

91262 Apply calculus methods in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply calculus methods in solving problems.	Apply calculus methods, using relational thinking, in solving problems.	Apply calculus methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area ( ). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) A function f is given by: $f(x) = 4x^3 - 2x^2 - 7x + 4$

Use calculus to find the gradient of the graph of the function at the point where $x = 3$

(b) For the cubic function $f(x) = \frac{1}{2}x^3 + \frac{1}{2}x$ find the equation of the tangent to the curve at $x = 2$

(c) The number of daily viewers for a new presenter on a video gaming streaming service can be modelled by the following equation:

$$V = -11t^2 + 528t \ \{0 \leq t \leq 48\}$$

Where V represents the number of daily viewers and t represents time in months.

(i) Find the rate(s) of change of daily viewers, with respect to t, when the number of daily viewers is 3520.

(ii) What is the maximum number of daily viewers that this new presenter gets during the first 48 months?

(iii) The number of daily viewers for an experienced presenter on a video gaming streaming service can be modelled by the following equation:

$$V = 1.6t^3 - 130t^2 + 2900t \ \{0 \leq t \leq 48\}$$

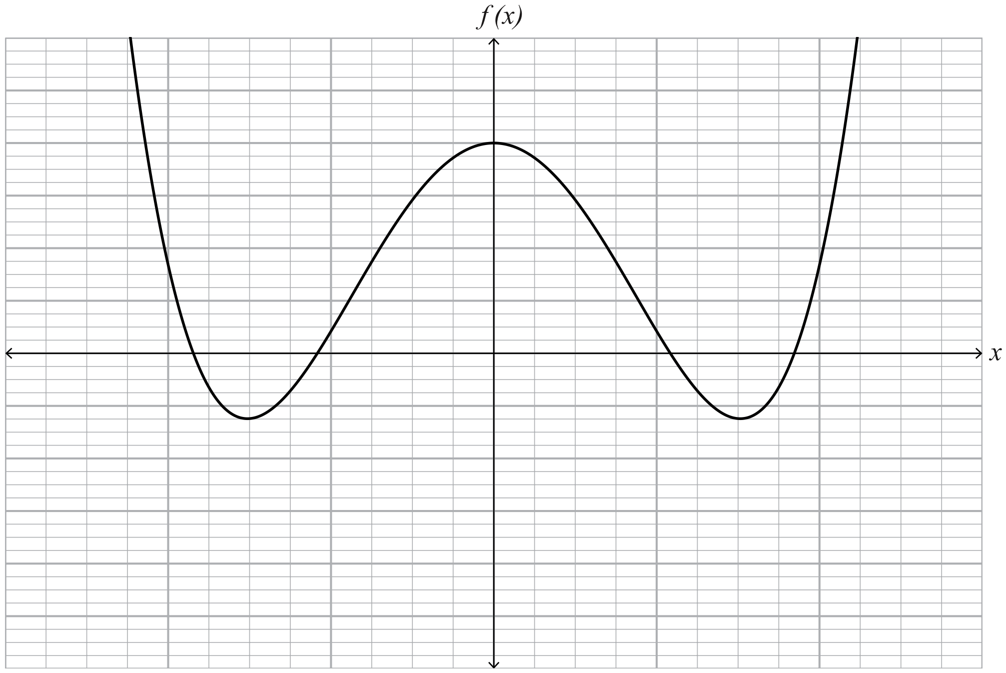
Where V represents the number of daily viewers and t represents time in months.

Once the stream reaches 10 000 daily viewers, it becomes monetised (the presenter earns money). If the daily viewership falls below 10 000, the presenter will lose this income and will no longer make any money.

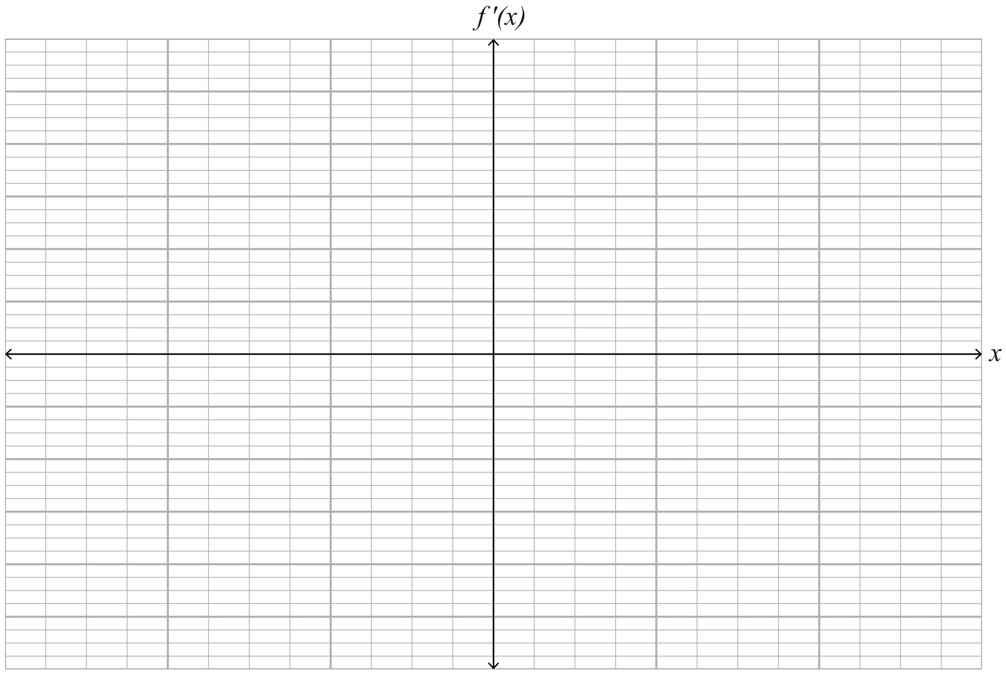
Use calculus methods to determine if the stream, after it becomes monetised, ever stops earning money for the presenter.

QUESTION TWO

(a) The graph of a function $y = f(x)$ is shown on the axes below.



Sketch the graph of the gradient function $y = f'(x)$ on the axes below.
Both sets of axes have the same horizontal scale.



If you need to redraw this graph, use the grid on page 12.

(b) The function f is given by: $f(x) = 5 + 3x + cx^2 - 2x^3$

At the point on the graph of the function where $x = 2$, the gradient is -5.

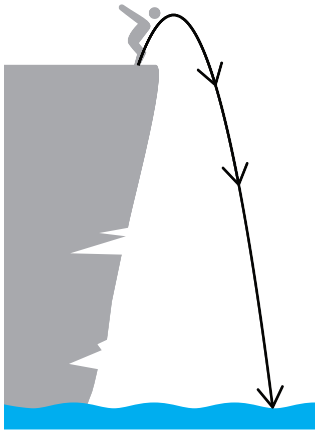
Find the value of c.

(c) (i) A cliff diver jumps up into the air above a cliff and then falls down into the water below.

Their acceleration is constant at -9.8 m s^{-2}.

The diver jumps up with an initial vertical velocity of 2.8 m s^{-1}.

Using calculus methods, find the velocity of the diver one second after they jumped.



(ii) The diver hits the water at a velocity of -22.68 m s^{-1} (negative velocities indicate the diver is moving down).

Find the maximum height (above the water) that the diver reached.

You may wish to begin by finding the height of the cliff above the water.

QUESTION THREE

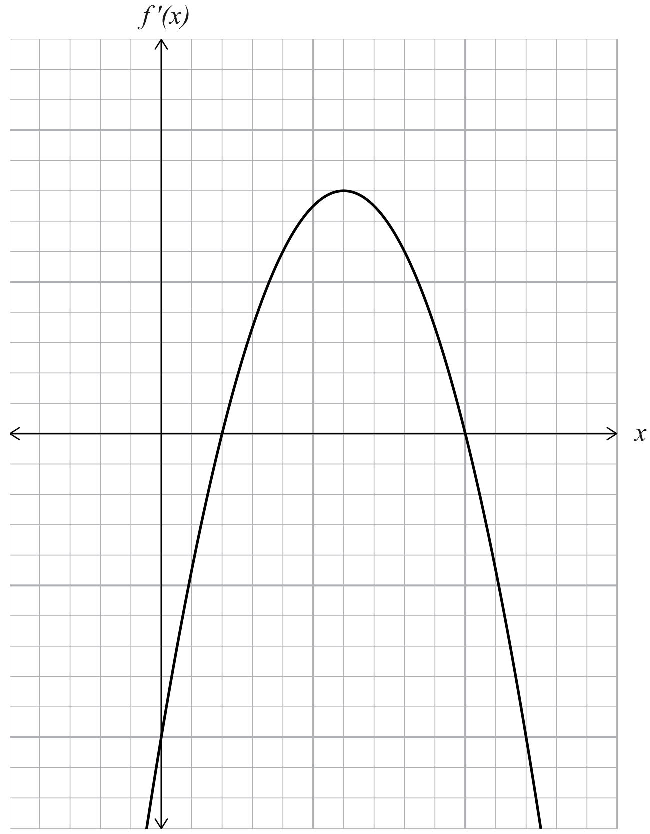
(a) A curve passes through the point $(1, 2.5)$ and has the derivative function: $f'(x) = 6x^2 + 5x - 1$

Find the co-ordinates of the point on the curve $y = f(x)$ where $x = 2$.

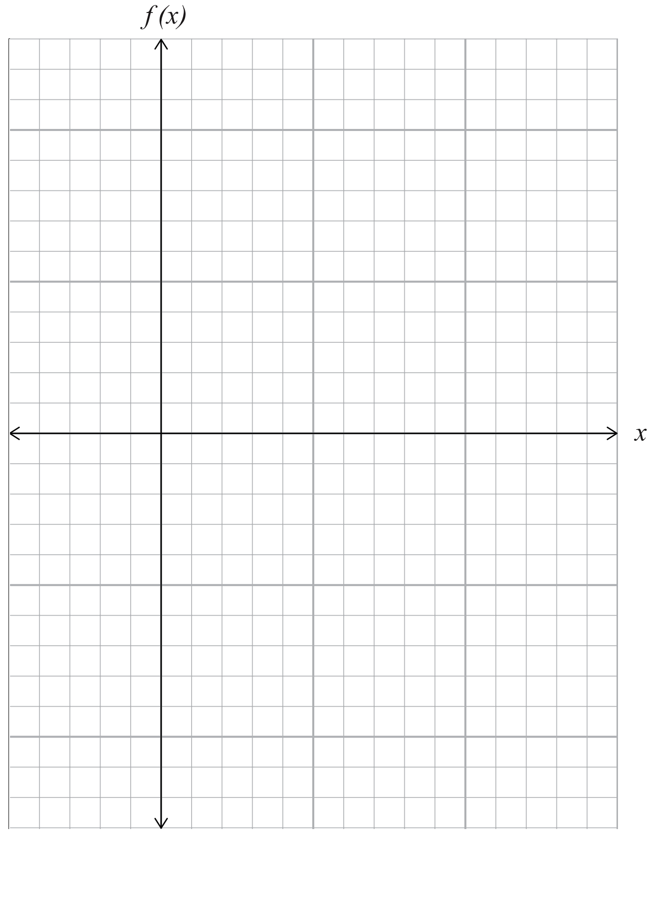
(b) An electric scooter is travelling down a straight footpath. Its velocity t seconds after passing a sign is given by $v(t) = 0.3t^2 + 1 \text{ m s}^{-1}$.

How far is the scooter from the sign when $t = 3$ seconds?

(c) The diagram below shows the graph of a gradient function $f'(x)$.



On the axes below sketch the graph of the function $f(x)$.

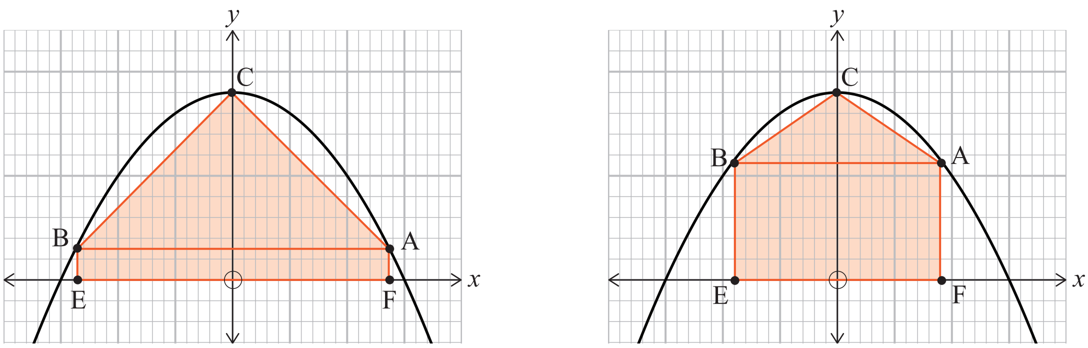


If you need to redraw this graph, use the grid on page 13.

(d) Two possible configurations of a house shape drawn below a parabola are shown below.

A house shape is drawn as shown where:

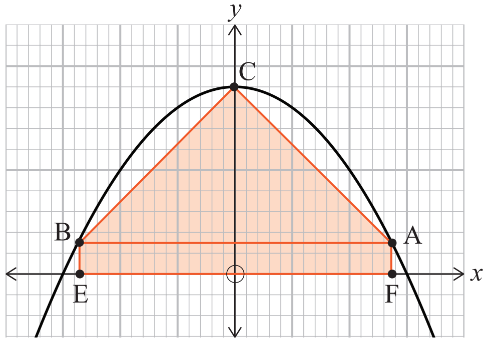
- C is at (0,9).
- A and B are points on the parabola $y = 9 - x^2$.
- E and F are on the x-axis.



(i) Find the height of the wall (AF or BE) when the area of the house shape is a maximum. Justify that this is the maximum area.

(ii) For the generalised problem where the house shape is bounded by a parabola where:

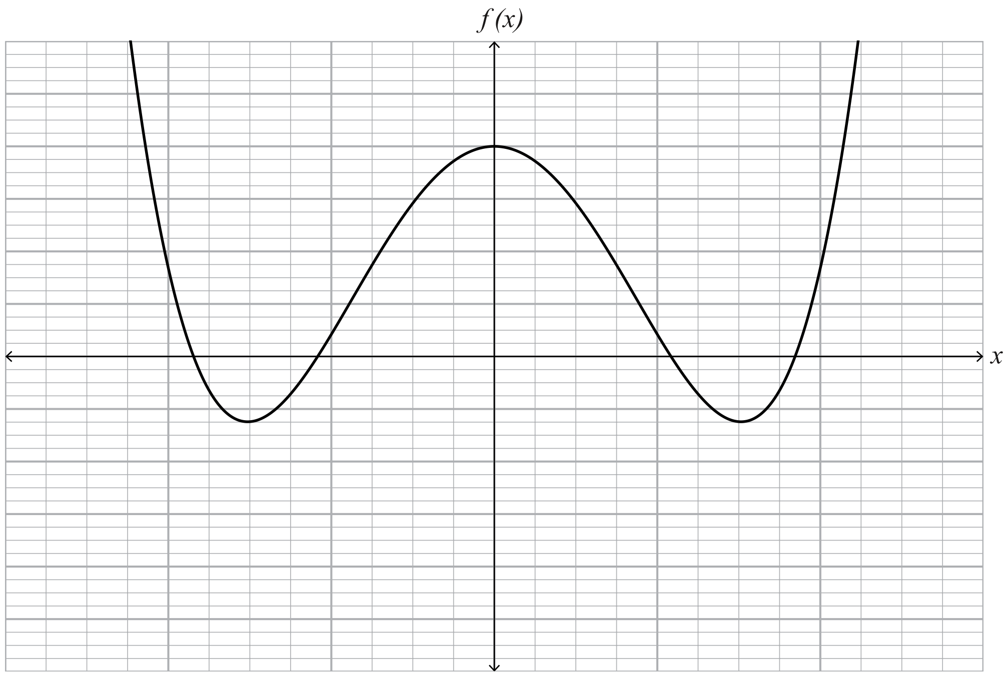
- C is at $(0,d)$.
- A and B are on the parabola $y = d - kx^2$.
- E and F are on the x-axis.



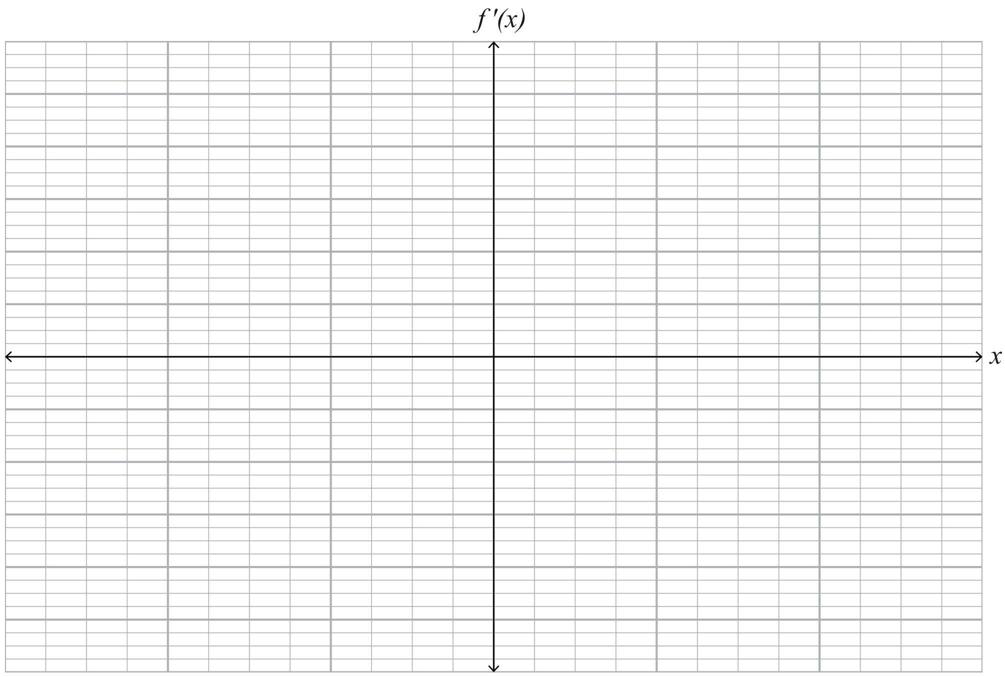
Show that the maximum area enclosed by the house shape occurs when the area of the rectangular section ABEF and the area of the triangular section ABC are equal.

SPARE GRIDS

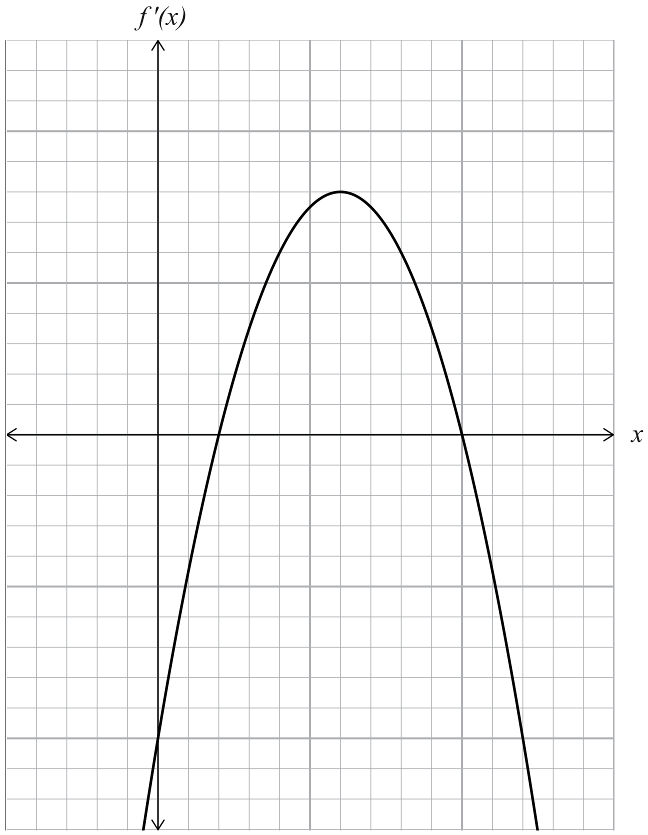
If you need to redo Question Two (a), use the grid below. You should make it clear which answer you want marked.



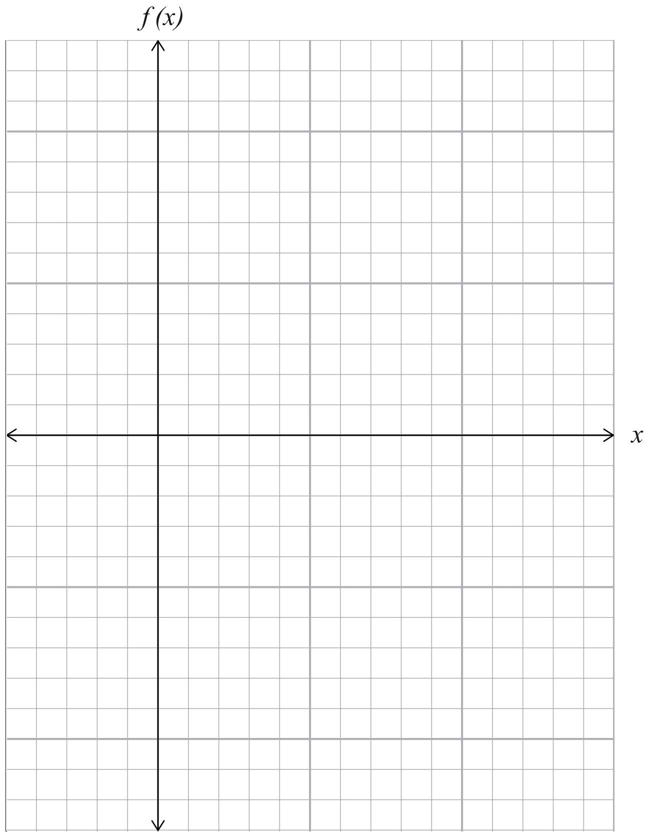
Sketch the graph of the gradient function $y = f'(x)$ on the axes below.
Both sets of axes have the same horizontal scale.



If you need to redo Question Three (c), use the grid below. You should make it clear which answer you want marked.



On the axes below sketch the graph of the function $f(x)$.



Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

91262