Assessment Schedule – 2023

Mathematics and Statistics: Apply probability methods in solving problems (91267)

Evidence

Q	Evidence	Achievement	Merit	Excellence
ONE (a)(i)	P(drops on 1st and wins 2nd game) $= 0.15 \times \frac{1}{9} = 0.0167$ Allow CAO.	• Probability correct.		
(ii)	P(wins 3rd game) = P(D,D,W) $0.15 \times 0.15 \times \frac{1}{8}$ + P(D,L,W) $0.15 \times 0.7389 \times \frac{1}{8}$ + P(L,D,W) $= 0.75 \times 0.15 \times \frac{1}{8}$ $= 0.00281 + 0.01385 + 0.014 = 0.0307 = \frac{59}{120}$ Accept any rounding.	• Consistent probability without changing P(winning) to $\frac{1}{8}$. OR One correct partial probability.	• Correct probability, i.e. adding all three possibilities.	
(iii)	P(wins in 4 attempts) = P(D,D,D,W) + P(D,D,L,W) + P(D,L,D,W) + P(L,D,D,W) = $0.15 \times 0.15 \times 0.15 \times \frac{1}{7} + 0.15 \times 0.15 \times 0.725 \times \frac{1}{7} +$ $0.15 \times 0.739 \times 0.15 \times \frac{1}{7} + \frac{3}{4} \times 0.15 \times 0.15 \times \frac{1}{7}$ $= 0.0004821 + 0.00233 + 0.002375 + 0.00241$ $= 0.007598$ P(wins in 3 attempts) = 0.307 (using answer from part (ii)) P(wins in 2 attempts) = $0.15 \times \frac{1}{9} + 0.75 \times \frac{1}{9} = 0.10$ P(wins in 1 attempt) = 0.10 (P(wins in max of 4 attempts) = P(win in 1) + P(win in 2) + P(win in 3) +P(win in4) = 0.10 + 0.10 + 0.0307 + 0.0076 = 0.2383		• P(wins in exactly four attempts) found (0.0076).	T1: Correct probability.
(b)(i)	P(winning claw game) = $0.7 \times 0.10 + 0.3 \times 0.25 = 0.145$ $= \frac{29}{200} = 14.5\%$	• Correct probability.		

(ii)	P(treat) = $0.7 \times 0.10 \times 6x + 0.7 \times 0.9 \times 3x$ $+ 0.3 \times 0.25 \times 2x + 0.3 \times 0.75 \times x = 0.40$ $2.685x = 0.40$ $x = 0.1489 = \frac{80}{537}$ P(friends and win claw and treat) $= 0.7 \times 0.1 \times 0.8934 = 0.063$	• CAO for either x or probability.	• Correct equation set up. OR x, $2x$, x correctly shown on tree.	T1: Correct x value found (prob of treat if lose claw by herself) with valid working. OR Final probability evaluated, with one error in algebraic set-up. T2: Correct probability found with valid working.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	A valid attempt at one question.	1 u	2 u	3 u	1r	2r	T1	T2 or 2T1

Figure 1: Probability tree for Question One (a)

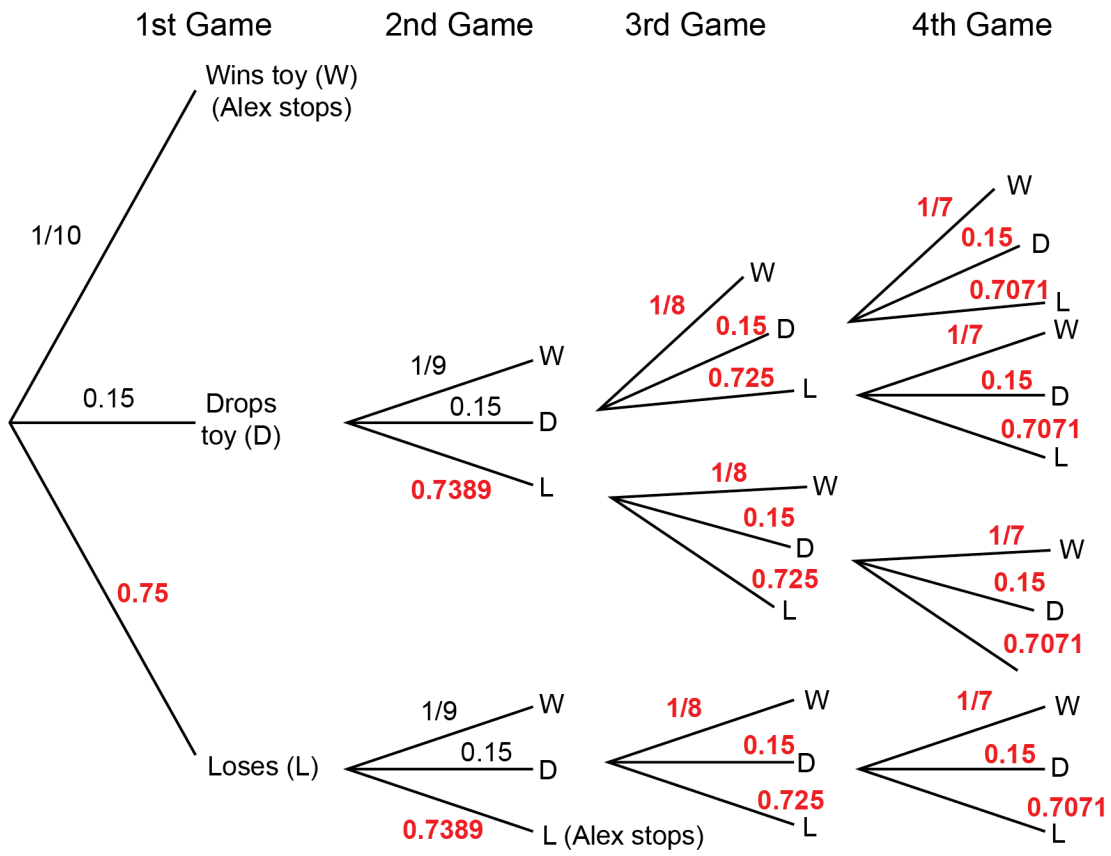
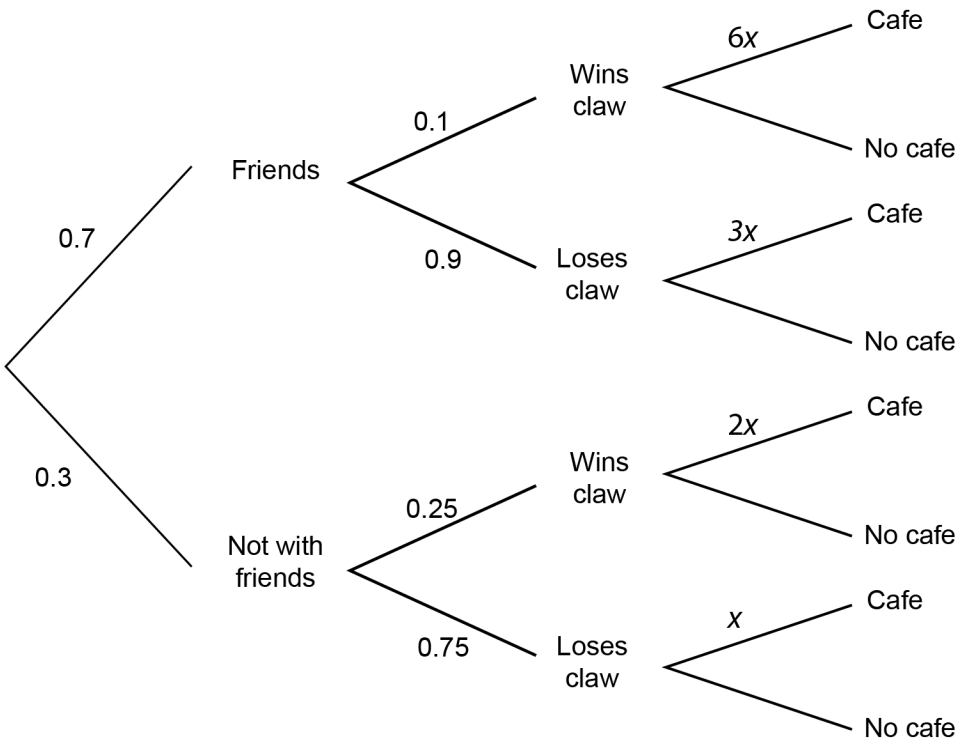


Figure 1: Probability tree for Question One (b)



Q	Evidence	Achievement	Merit	Excellence
TWO (a)(i)	P(NZ youth and watched streaming service) $= \frac{191}{1420} = 0.1345$	• Correct probability.		
(ii)	P(online video if youth) $= \frac{228}{251} = 0.908$ P(online video if adult) $= \frac{610}{1169} = 0.522$ Possible justifications include: Tao is not correct, as he only looked at the raw numbers, not the probabilities. Youth are more likely to watch videos online than adults since 0.908 > 0.522. By considering the relative risks.	• One correct conditional probability.	• Both conditional probabilities correct. AND Conclusion based on comparison.	
(iii)	P(online video) $= \frac{838}{1420} = 0.5901 = 59\%$ P(streaming services) $= \frac{724}{1420} = 0.5099 = 51\%$ P(TV) $= \frac{795}{1420} = 0.5599 = 56\%$	• Three correct percentages found.		
(iv)	P(streaming if youth) $= \frac{191}{251} = 0.7610$ P(streaming if adult) $= \frac{533}{1169} = 0.4559$ RR = $= \frac{0.7610}{0.4559} = 1.669$ which is close to 1.70. 1.67 times as likely indicates youth are 67% (nearly 70%) more likely to watch streaming services than adults. P(TV if youth) $= \frac{90}{251} = 0.3586$ P(TV if adults) $= \frac{705}{1169} = 0.6031$ RR $= \frac{0.3586}{0.6031} = 0.5946$, which is close to 0.6, indicating 0.59 times as likely or 41% less likely for youth to watch TV than adults. Therefore both claims are valid / correct, as they are "about" the same. Do not accept claims as invalid.	• One correct probability.	• One relative risk found and interpreted. OR Multiplicative reasoning used to evaluate at least one claim.	T1: Both relative risks calculated but only one interpreted correctly. T2: Both relative risks calculated and interpreted AND claim evaluated as correct.

(b)(i)	P(streaming services in 2016) $= \frac{102}{249} = 0.4096$ P(streaming services in 2022) $= \frac{473}{706} = 0.6700$ 0.6700 – 0.4096 = 0.2604 The percentage of youth watching streaming services in 2022 is 26% higher than in 2016. (Do NOT accept use of relative risk of 1.63, as it is 63% more likely NOT 63% higher)	• Correct difference **in percentages** found.		
(ii)	**2022** P(streaming services in youth) = $\frac{473}{706} = 0.67$ Expected value = 0.67 × 400 = 267.99, so expect 268. **2021** P(streaming services in youth) $= \frac{191}{251} = 0.761$ Expected value = 0.761 × 400 = 304.4, so expect 304. **2020** P(streaming services in youth) $= \frac{181}{262} = 0.69$ Expected value 0.69 × 400 = 276.3, so expect 276. Candidate could, for example, justify they expect the drop from 2021 to 2022 to continue, so estimate probability to be as low as 0.58, or for it to return to 2021 at nearly 0.80. Accept an expected number between 230 and 320 with valid working or justification.	• One expected value found from the table for any year. OR At least TWO probabilities calculated.	• Expected value for 2023 estimated with working or justification based on at least one probability from the table. (Must be whole number.)	

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	A valid attempt at one question.	1 u	2 u	3 u	1r	2r	T1	T2

Q	Evidence	Achievement	Merit	Excellence
THREE (a)(i)	Normal distribution $\mu = 113$ $\sigma = 35$ $P(X < 92) = P(Z < -0.6) = 0.2743$ (0.2742 from tables) Allow CAO	• Probability correct.		
(ii)	$P(120 < X < 180) = P(0.2 < Z < 1.914) = 0.3929$ (0.3928 from tables) Allow CAO	• Probability correct.		
(b)(i)	Inverse normal $P(x_1 < X < x_2) = 0.5$ $P(0 < Z < z_2) = 0.25$ $z_2 = 0.674$ $z_1 = -0.674$ $94 \pm 0.674 \times 30$ Or GC with area: centre = 0.50 $x_1 = 73.8$ $x_2 = 114.2$ IQR = 114.2 – 73.8 = 40.4 Middle 50% of youth spend between 73.8 and 114.2 minutes watching online videos daily.	• LQ or UQ found. OR CAO.	• Correct IQR. OR Both quartiles found and interpreted in context. Some evidence of working or diagram must be shown.	
(ii)	Streaming services: $P(X > 120) = P(Z > 0.2)$ Expected number = 50 × 0.4207 = 21.035 = 21 Online videos: $P(X < 120) = P(Z > 0.867) = 0.1931$ Expected number = 50 × 0.1931 = 9.655 = 10 Then difference will be 21 – 10 = 11. Would expect 11 (or 12) more youth respondents to have spent over 120 minutes watching streaming services than watching online videos.	• One probability found (i.e. either 0.4207 or 0.1931) OR One relevant expected value found. OR CAO	• Difference in two expected values found with valid evidence. • Must be a whole number.	

(iii)	Normal distribution not appropriate because: • $P(X < 0) \neq 0$ (can't have negative minutes, so distribution needs to be truncated). • There would be some youth who do not spend any time watching videos or on streaming services, which would give a cluster on left tail which would not fit a unimodal normal distribution shape with peak at the centre. • The shape is unlikely to be symmetrical or bell-shaped, as some students could spend a very long time on these devices, making it right-skewed, which is not normally distributed. • Programmes on streaming services are often in multiples of 30 or 60 minutes, so may not be continuous, but have clusters at common episode / movie length times.	• One valid reason given for one context, which must link to a normal distribution model.	• Two valid reasons given which must link to a normal distribution model.	
(c)(i)	A large number of youth not watching any TV (0 minutes) would have brought down the mean and median significantly, and caused a bimodal or skewed distribution.		• Valid discussion in part (i). AND	T1: Correct standard deviation found with valid working.
(ii)	$P(X > 150) = 0.12$ $P(Z > 1.175) = 0.12$ $Z = \frac{X - \mu}{\sigma}$ $1.175 = \frac{150 - 114}{\sigma}$ $\sigma = 30.6$ minutes	• CAO for standard deviation. OR Correct z-score.	Correct z-score.	T2: Standard deviation found. AND Discussion of how normal distribution shape AND mean would be affected by 0-values.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	A valid attempt at one question.	1 u	2 u	3 u	1r	2r	T1	T2

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 13	14 – 19	20 – 24