Assessment Schedule – 2013

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Assessment Criteria

Achievement	Merit	Excellence
Apply the algebra of complex numbers in solving problems must involve: • using a range of appropriate algebraic methods • demonstrating knowledge of concepts and terms • and communicating using appropriate representations.	*Apply the algebra of complex numbers, using relational thinking, in solving problems* must involve one or more of: • carrying out a logical sequence of steps • connecting different concepts and representations • demonstrating understanding of concepts • forming and using a model • and relating findings to a context, or communicating thinking using appropriate mathematical statements.	*Apply the algebra of complex numbers, using extended abstract thinking, in solving problems* must involve one or more of: • devising a strategy to investigate or solve problem • identifying relevant concepts in context • developing a chain of logical reasoning, or proof • forming a generalisation • and using correct mathematical statements, or communicating mathematical insight.

Evidence Statement

One	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$-8-4\sqrt{5}$	Or equivalent.		
(b)	$-4+4\text{i}$ clearly marked on Argand diagram.	Must plot point.		
(c)	$z_1 = 2-3\text{i} \quad \Rightarrow \quad z_2 = 2+3\text{i}$ $\left(z-2-3\text{i}\right)\left(z-2+3\text{i}\right)$ $= z^2-4z+13$ $(z^2-4z+13)(z+k) = z^3-4z^2+13z+kz^2-4kz+13k$ $= z^3+(k-4)z^2+(13-4k)z+13k$ $=0$ $\Rightarrow k=4$ $\therefore \text{p}=52, \text{roots } 2+3i,\ 2-3i, -4$	Either one other root found or $p = 52$	Both other roots and p found. CRO	
(d)	$w = \dfrac{1}{1+\text{i}}+\text{i}$ $= \dfrac{\left(1-\text{i}\right)}{\left(1+\text{i}\right)\left(1-\text{i}\right)}+\text{i}$ $= \dfrac{\left(1-\text{i}\right)}{2}+\text{i}$ $= \dfrac{1+\text{i}}{2}$ $= \dfrac{1}{2}+\dfrac{1}{2}\text{i}$ Arg $(w) = \dfrac{\pi}{4}$ or $45°$	Simplified expression for w or Arg $(w) = 0.785$	Exact value of argument found. CRO.	

One	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(e)	$uv = (6+ki)(4+ki)$ $= 24 + 10ki - k^2$ $\text{Arg} = \frac{\pi}{4} \Rightarrow Re(uv) = Im(uv)$ $24 - k^2 = 10k$ $k^2 + 10k - 24 = 0$ $(k+12)(k-2) = 0$ $k = -12$ or 2 $k = -12 \Rightarrow uv = -120 - 120i$ $\therefore \text{Arg} = \frac{-3\pi}{4} \neq \frac{\pi}{4}$ $\therefore k \neq -12$ $k = 2 \Rightarrow uv = 20 + 20i$ $\therefore \text{Arg} = \frac{\pi}{4}$ $\therefore k = 2$	Expanded expression for *uv*. *CRO*	Two correct solutions for k from quadratic equation.	Correct answer ($k = 2$) with a logical chain of reasoning.

NØ = No relevant evidence	**A3** = 2 u	**M5** = 1 r	**E7** = 1 t with 1 minor error
N1 = 1 partial solution	**A4** = 3 u	**M6** = 2 r	**E8** = 1 t
N2 = 1 u			

Two	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	–7	Remainder found.		
(b)	$\frac{1}{3}\text{cis}\left(\frac{\pi}{6}\right)$	Or equivalent.		
(c)	$\alpha+\beta=-9$ $3\alpha+3\beta=-27$ $\alpha\beta=-12$ $3\alpha\cdot 3\beta=9\alpha\beta=-108$ $x^2+27x-108=0$	Two correct terms out of x^2, $27x$ or -108.	A correct equation.	
(d)	$au^2+bu+c=0$ $a(x+\text{i}y)^2+b(x+\text{i}y)+c=0$ $ax^2+2axy\text{i}-ay^2+bx+by\text{i}+c=0$ (3) $ax^2-ay^2+bx+c+(2axy+by)\text{i}=0$ so $ax^2-ay^2+bx+c=0$ and $2axy+by=0$ $a\bar{u}^2+b\bar{u}+c$ $=a(x-\text{i}y)^2+b(x-\text{i}y)+c$ $=ax^2-2axy\text{i}-ay^2+bx-by\text{i}+c$ (8) $=ax^2-ay^2+bx+c-(2axy+by)\text{i}$ $=0-0$ $=0$ as required	Expanded expression for equation in terms of x and y (ie 3rd line).	Proof correct with logical mathematical statements.	
(e)	$\frac{z+2\text{i}}{z-2\text{i}}=\frac{x+\text{i}y+2\text{i}}{x+\text{i}y-2\text{i}}$ $=\frac{(x+(y+2)\text{i})}{(x+(y-2)\text{i})}\cdot\frac{(x-(y-2)\text{i})}{(x-(y-2)\text{i})}$ $=\frac{x^2-xy\text{i}+2x\text{i}+xy\text{i}+2x\text{i}+y^2-4}{x^2+(y-2)^2}$ (3) $=\frac{x^2+y^2-4+4x\text{i}}{x^2+(y-2)^2}$ As purely imaginary, the real part equals zero. $x^2+y^2-4=0$ or $x^2+y^2=4$ Locus is a circle, radius 2, centred at (0,0). [excludes the point (0,2)]	*CRO*	Expanded expression with correct numerator or denominator. (ie 3rd line).	Correct answer with a logical chain of reasoning. Do not have to check when denominator equal to zero to find exclusion.

NØ = No relevant evidence	**A3** = 2 u	**M5** = 1 r	**E7** = 1 t with 1 minor error
N1 = 1 partial solution	**A4** = 3 u	**M6** = 2 r	**E8** = 1 t
N2 = 1 u			

Three	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$-3 \pm \sqrt{11}\,\mathrm{i}$	Simplified solutions.		
(b)	$p = 3+4\mathrm{i}$ $\bar{q} = 2+3\mathrm{i}$ $p\bar{q} = -6+17\mathrm{i}$	Simplified solution.		
(c)	$\sqrt{x} - 3 = \sqrt{x-p}$ $x - 6\sqrt{x} + 9 = x - p$ $6\sqrt{x} = p + 9$ (3) $36x = (p+9)^2$ $x = \frac{(p+9)^2}{36}$ or $x = \frac{p^2+18p+81}{36}$ or $x = \frac{p^2}{36} + \frac{p}{2} + \frac{9}{4}$	Correct expression with x's eliminated (ie 3rd line).	Or equivalent.	
(d)	$z^3 = -n$ $z^3 = n\,\mathrm{cis}\,\pi$ $z_1 = \sqrt[3]{n}\,\mathrm{cis}\left(\frac{\pi}{3}\right)$ $z_2 = \sqrt[3]{n}\,\mathrm{cis}\,\pi$ $z_3 = \sqrt[3]{n}\,\mathrm{cis}\left(\frac{5\pi}{3}\right) = \sqrt[3]{n}\,\mathrm{cis}\left(\frac{-\pi}{3}\right)$ Accept arguments in decimal radians: 1.047, 3.142, 5.236 or –1.047 Accept arguments in degrees: 60°, 180°, 300° or – 60°	ONE correct solution.	THREE correct solutions.	
(e)	$6 + x = 4\sqrt{3x+k}$ $x^2 + 12x + 36 = 16(3x+k)$ $x^2 - 36x + 36 - 16k = 0$ (3) No real roots means $b^2 - 4ac < 0$ $(-36)^2 - 4(36-16k) < 0$ $1296 - 144 + 64k < 0$ $64k < -1152$ $k < -18$	CRO	Simplified quadratic expression (3rd line).	Correct answer with a logical chain of reasoning.

NØ = No relevant evidence	**A3** = 2 u	**M5** = 1 r	**E7** = 1 t with 1 minor error
N1 = 1 partial solution	**A4** = 3 u	**M6** = 2 r	**E8** = 1 t
N2 = 1 u			

Judgement Statement

	Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
Score range	0 – 7	8 – 12	13 – 18	19 – 24