Assessment Schedule – 2014

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

Q1	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$p = \frac{1}{2}$	OR equivalent.		
(b)	$u^4 = \left[\sqrt{18}\text{cis}\frac{-\pi}{4}\right]^4$ $= 324\text{cis}(-\pi)$ or $324\text{cis}(\pi)$ $u^4 = \left(4.24\text{cis}(-45°)\right)^4$ $= 324\text{cis}(-180°)$ OR $324\text{cis}(180°)$	OR equivalent.		
(c)	$x - 3 = \sqrt{33 - 4x}$ $(x-3)^2 = 33 - 4x$ $x^2 - 2x - 24 = 0$ $x = 6$ OR $x = -4$ CHECK: $x = 6$ $\quad 6 = \sqrt{9} + 3$ TRUE CHECK: $x = -4$ $\quad -4 = \sqrt{49} + 3$ FALSE Therefore there is one solution: $x = 6$	$x = 6$ and -4 both given as solutions. OR $x = 6$ CRO	$x = 6$ chosen after discarding $x = -4$	
(d)	$z^4 = -4k^2\text{i} = 4k^2\text{cis}\left(\frac{-\pi}{2}\right)$ OR $4k^2\text{cis}(-90°)$ $z_1 = \sqrt{2k}\text{cis}\left(\frac{-\pi}{8}\right)$ OR $z_1 = \sqrt{2k}\text{cis}\left(\frac{15\pi}{8}\right)$ $z_2 = \sqrt{2k}\text{cis}\left(\frac{3\pi}{8}\right)$ OR $z_2 = \sqrt{2k}\text{cis}\left(\frac{-13\pi}{8}\right)$ $z_3 = \sqrt{2k}\text{cis}\left(\frac{7\pi}{8}\right)$ OR $z_3 = \sqrt{2k}\text{cis}\left(\frac{-9\pi}{8}\right)$ $z_4 = \sqrt{2k}\text{cis}\left(\frac{11\pi}{8}\right)$ or $z_4 = \sqrt{2k}\text{cis}\left(\frac{-5\pi}{8}\right)$	One correct solution. OR 4 correct arguments in degrees or radians.	All correct solutions given.	
(e)	Let $z = x + \text{i}y$ $\lvert x + \text{i}y - 1 + 2\text{i}\rvert = \lvert x + \text{i}y + 1\rvert$ $\lvert(x-1) + (y+2)\text{i}\rvert = \lvert(x+1) + y\text{i}\rvert$ $\sqrt{(x-1)^2 + (y+2)^2} = \sqrt{(x+1)^2 + y^2}$ $(x-1)^2 + (y+2)^2 = (x+1)^2 + y^2$ $x^2 - 2x + 1 + y^2 + 4y + 4 = x^2 + 2x + 1 + y^2$ $-2x + 4y + 5 = 2x + 1$ $4y - 4x + 4 = 0 \qquad y - x + 1 = 0$ or $y = x - 1$	Correct 3rd line.	Correct 5th line. $(x-1)^2 + \ldots$	Correct expression for locus.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	1 partial solution	1u	2u	3u	1r	2r	1t with 1 minor error	1 t

Q2	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$x = 2 \pm 2\sqrt{3}\mathrm{i}$ or $x = 2 \pm \sqrt{12}\mathrm{i}$	Correct solution.		
(b)	$w = 2\operatorname{cis}\frac{\pi}{3}$ $w^4 = 16\operatorname{cis}\frac{4\pi}{3}$ OR $w^4 = 16\operatorname{cis}(240°)$ $w^4 = -8 - 8\sqrt{3}\,\mathrm{i}$ OR $w^4 = -8 - 13.86\mathrm{i}$	OR Equivalent rectangular form.		
(c)	$w_1 = 2 - 3\mathrm{i}$ $w_2 = 2 + 3\mathrm{i}$ $\left[w-(2-3\mathrm{i})\right]\left[w-(2+3\mathrm{i})\right]$ $= w^2 - 4w + 13$ $\Rightarrow g(x) = (w^2 - 4w + 13)(kw + c)$ $\Rightarrow k = 3$ $c = -2$ Coefficient of w $-4c + 13k$ $= 47$ $\Rightarrow A = 47$ OR $A = \frac{94 - 141\mathrm{i}}{2 - 3\mathrm{i}}$ $w_3 = \frac{2}{3}$	$w^2 - 4w + 13$ OR $A = 47$ OR $w_2 = 2 + 3\mathrm{i}$ and $w_3 = \frac{2}{3}$ OR CRO	Correct value of A and other 2 solutions correct. $A = 47$ $w_2 = 2 + 3\mathrm{i}$ $w_3 = \frac{2}{3}$	
(d)(i) (ii)	Circle centre (3,4), radius 2 5	Correct locus drawn.	Correct locus drawn and correct maximum value.	
(e)	$\frac{1+3\mathrm{i}}{p+p\mathrm{i}} = \frac{p - q\mathrm{i} + 3p\mathrm{i} + 3q}{p^2 + q^2}$ $= \frac{p + 3q + (3p - q)\mathrm{i}}{p^2 + q^2}$ $\operatorname{Arg}(z) = \frac{\pi}{4} \rightarrow \tan(\operatorname{Arg}(z)) = 1$ $\operatorname{Re}(z) = \operatorname{Im}(z)$ if $\operatorname{Arg}(z) = \frac{\pi}{4}$ $\tan\left(\frac{\pi}{4}\right) = \frac{3p - q}{p + 3q}$ $\frac{3p - q}{p + 3q} = 1$ $p - 2q = 0$	Correct expression without i^2 (1st line).	$\operatorname{Arg}(z) = \frac{\pi}{4}$ interpreted to give a correct relationship such as $\tan(\operatorname{Arg}(z)) = \frac{\operatorname{Im}(z)}{\operatorname{Re}(z)}$	Correct solution.

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No response; no relevant evidence.	1 partial solution	1u	2u	3u	1r	2r	1t with 1 minor error	1 t

Q3	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$25-16\sqrt{3}$	Correct expression.		
(b)	$r=-8-5\text{i}$ marked correctly on diagram.	Correct point indicated on diagram.		
(c)	$px^2-4px+1=0$ has no real solutions $\rightarrow b^2-4ac<0$ $16p^2-4p<0$ $4p(4p-1)<0$ $0<p<\frac{1}{4}$	Correct inequation (2nd line). OR CRO	Correct solution.	
(d)	$\bar{z}^2+\frac{1}{z^2}=(3-2\text{i})^2+\frac{1}{(3+2\text{i})^2}$ $=5-12\text{i}+\frac{1}{5+12\text{i}}$ $=5\frac{5}{169}-12\frac{12}{169}\text{i}$ OR $\frac{850}{169}-\frac{2040}{169}\text{i}$ OR $5.03-12.07\text{i}$	Correct expansion (2nd line). OR CRO.	Correct expression.	
(e)(i) (ii)	Roots $\alpha,\beta,\gamma \Rightarrow (x-\alpha)(x-\beta)(x-\gamma)=0$ $x^3-(\alpha+\beta+\gamma)x^2+(\alpha\gamma+\beta\gamma+\alpha\beta)x-\alpha\beta\gamma=0$ Since $ax^3+bx^2+cx+d=0$ $x^3+\frac{b}{a}x^2+\frac{c}{a}x+\frac{d}{a}=0$ Comparing coefficients of x^2: $\alpha+\beta+\gamma=\frac{-b}{a}$ of x : $\alpha\beta+\beta\gamma+\alpha\gamma=\frac{c}{a}$ of constant term: $\alpha\beta\gamma=\frac{-d}{a}$ $\alpha^2\beta\gamma+\alpha\beta^2\gamma+\alpha\beta\gamma^2=\alpha\beta\gamma(\alpha+\beta+\gamma)$ $=\frac{-d}{a}\times\frac{-b}{a}$ $=\frac{bd}{a^2}$	One correct proof.	3 correct proofs.	All 4 proofs correct.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	1 partial solution	1u	2u	3u	1r	2r	1t with 1 minor error	1 t

Cut Scores

	Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
Score range	0 – 6	7 – 12	13 – 19	20 – 24