Assessment Schedule – 2015

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence

Q1	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	$x = 4 \pm 2\sqrt{3}$	Correct solution.		
(b)	$u^3 = -8$ clearly marked on Argand diagram.	Correct solution.		
(c)	$p(3-7\text{i}) + q(-4+6\text{i}) = 6.5 - 11\text{i}$ $3p - 7p\text{i} - 4p + 6\text{i}q = 6.5 - 11\text{i}$ $\Rightarrow 3p - 4q = 6.5$ and $-7p + 6q = -11$ $p = 0.5$ $q = -1.25$	Correct second line.	Correct solution.	
(d)	$'b^2 - 4ac' = (2c+1)^2 + 4 \times 3 \times (c+3)$ $= 4c^2 + 16c + 37$ *OR* $= 4\left(c^2 + 4c\right) + 37$ $= (2c+4)^2 + 21 \geq 0$ $= 4\left(\left[c+2\right]^2 - 4\right) + 37$ Since $(2c+4)^2 : \geq 0$ $= 4\left(c+2\right)^2 + 21$ Which is always positive, as $\left(c+2\right)^2$ is always positive.	Correct second line.	Correct solution.	

(e)	Let $x^2+bx+c=(x-p)(x-q)$ $=x^2-qx-px+pq$ $\Rightarrow b=-p-q$ and $c=pq$ Let $x^2+dx+e=(x-p)(x-r)$ $=x^2-rx-px+pr$ $\Rightarrow d=-p-r$ and $e=pr$ $\therefore \dfrac{e-c}{b-d} = \dfrac{pr-pq}{-p-q+p+r}$ $=\dfrac{p(r-q)}{-q+r}=p$	The two 'let' lines.	Successfully equating coefficients.	Correct proof.
	OR $(x-p)$ is a factor of x^2+bx+c $\Rightarrow p^2+bp+c=0$ $(x-p)$ is a factor of x^2+dx+e $\Rightarrow p^2+dp+e=0$ $\Rightarrow p^2+bp+c=p^2+dp+e$ $bp+c=dp+e$ $bp-dp=e-c$ $p(b-d)=e-c$ $p=\dfrac{e-c}{b-d}$	EITHER $p^2+bp+c=0$ OR $p^2+dp+e=0$	Equating the two equations in the ACH column.	Correct proof.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	1 partial solution.	1u	2u	3u	1r	2r	1t with 1 minor error.	1 t

Q2	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	–23	Correct solution.		
(b)	$\frac{2+3\mathrm{i}}{5+\mathrm{i}}=\frac{(2+3\mathrm{i})(5-\mathrm{i})}{(5+\mathrm{i})(5-\mathrm{i})}$ $=\frac{13+13\mathrm{i}}{26}$ $=\frac{13}{26}(1+\mathrm{i})$ $=\frac{1}{2}(1+\mathrm{i})$ $\therefore k=\frac{1}{2}$	Correct solution.		
(c)	$1=A(x(x-1))+B(x-1)+Cx^2$ $=Ax^2-Ax+Bx-B+Cx^2$ $=(A+C)x^2+(B-A)x-B$ $\therefore B=-1, A=-1, C=1$	Correct second line.	Correct solution.	
(d)	$\left(\frac{4\mathrm{i}^7-\mathrm{i}}{1+2\mathrm{i}}\right)^2=\left(\frac{-4\mathrm{i}-\mathrm{i}}{1+2\mathrm{i}}\right)^2$ $=\left(\frac{-5\mathrm{i}}{1+2\mathrm{i}}\right)^2$ $=\frac{-25}{-3+4\mathrm{i}}$ $=\frac{-25(-3-4\mathrm{i}))}{(-3+4\mathrm{i})(-3-4\mathrm{i})}$ $=\frac{75+100\mathrm{i}}{25}$ $=3+4\mathrm{i}$	Correct third line. Or CRO	Correct solution.	
(e)	$\frac{z-2}{z+5}=\frac{x+y\mathrm{i}-2}{x+y\mathrm{i}+5}$ $=\frac{(x-2)+y\mathrm{i}}{(x+5)+y\mathrm{i}}$ $=\frac{[(x-2)+y\mathrm{i}][(x+5)-y\mathrm{i}]}{[(x+5)+y\mathrm{i}][(x+5)-y\mathrm{i}]}$ $=\frac{x^2+y^2+3x-10+7y\mathrm{i}}{x^2+10x+25+y^2}$ $\text{Arg}=\frac{\pi}{4}\Rightarrow x^2+y^2+3x-10=7y$ $\therefore\ x^2+y^2+3x-10-7y=0$	Correct second line.	Correct 'simplification', 4th line. Accept correct factorised denominator.	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	1 partial solution.	1u	2u	3u	1r	2r	1t with 1 minor error.	1 t

Q3	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	$zw = (4+2\text{i})(-1+3\text{i})$ $= -4+12\text{i}-2\text{i}+6\text{i}^2$ $= -10+10\text{i}$ $\text{Arg}(zw) = 135° = \frac{3\pi}{4} = 2.36 \text{ rad}$	Correct solution.		
(b)	$b^2 - 4ac = 0$ $\frac{1}{k^2} - 4k \times 2 = 0$ $\frac{1}{k^2} = 8k$ $k^3 = \frac{1}{8}$ $k = \frac{1}{2}$	Correct solution.		
(c)	$f(-2) = 0$ $\Rightarrow -24 + 4A + 16 = 0$ $A = 2$ $3w^3 + Aw^2 - 3w + 10 = (w+2)(3w^2 - 4w + 5)$ $\therefore (w+2)(3w^2 - 4w + 5) = 0$ $w = \frac{4 \pm \sqrt{16 - 4 \times 3 \times 5}}{6}$ $= \frac{4 \pm \sqrt{-44}}{6}$ $= \frac{2 \pm \sqrt{11}\text{i}}{3}$ Or w $0.667 \pm 1.1055i$	Correct value of A found.	Correct solution.	
(d)	$z^3 = 2k \text{cis}\left(\frac{\pi}{3}\right)$ or $\text{Arg} = 1.047$ or $60°$ Therefore roots are: $z_1 = \sqrt[3]{2k} \text{ cis}\left(\frac{\pi}{9}\right)$ or $\text{Arg} = 0.349$ or $20°$ $z_2 = \sqrt[3]{2k} \text{ cis}\left(\frac{7\pi}{9}\right)$ or $\text{Arg} = 2.443$ or $140°$ $z_3 = \sqrt[3]{2k} \text{ cis}\left(\frac{13\pi}{9}\right)$ or $\text{Arg} = 4.538$ or $260°$ OR $\sqrt[3]{2k} \text{ cis}\left(\frac{-5\pi}{9}\right)$ or $\text{Arg} = -1.745$ or $-100°$	One correct solution or 3 correct arguments.	Three correct solutions – allow alternative correct values.	

(e)(i)	$z^5 = 1$ $z^5 = 1\,\text{cis}\,0$ Therefore roots are $z_1 = 1\,\text{cis}\,0$ or 1 $z_2 = 1\,\text{cis}\left(\frac{2\pi}{5}\right)$ or $\text{Arg} = 1.257$ or $72°$ or $0.309 + 0.951\text{i}$ $z_3 = 1\,\text{cis}\left(\frac{4\pi}{5}\right)$ or $\text{Arg} = 2.513$ or $144°$ or $-0.809 + 0.588\text{i}$ $z_4 = 1\,\text{cis}\left(\frac{6\pi}{5}\right)$ or $\text{Arg} = 3.770$ or $216°$ or $1\,\text{cis}\left(\frac{-4\pi}{5}\right)$ or $\text{Arg} = -2.513$ or $-144°$ or $-0.809 - 0.588\text{i}$ $z_5 = 1\,\text{cis}\left(\frac{8\pi}{5}\right)$ or $\text{Arg} = 5.027$ or $288°$ or $1\,\text{cis}\left(\frac{-2\pi}{5}\right)$ or $\text{Arg} = -1.257$ or $-72°$ or $0.309 - 0.951\text{i}$		Correct values of z_1 to z_5.	Correct solution.
(ii)	Let $p = 1\,\text{cis}\left(\frac{2\pi}{5}\right)$ which is z_2 above or p 0.309 + 0.951i Then $p^2 = \left(1\,\text{cis}\left(\frac{2\pi}{5}\right)\right)^2 = 1\,\text{cis}\left(\frac{4\pi}{5}\right) = z_3$ $p^3 = \left(1\,\text{cis}\left(\frac{2\pi}{5}\right)\right)^3 = 1\,\text{cis}\left(\frac{6\pi}{5}\right) = z_4$ $p^4 = \left(1\,\text{cis}\left(\frac{2\pi}{5}\right)\right)^4 = 1\,\text{cis}\left(\frac{8\pi}{5}\right) = z_5$ And $z_1 = 1\,\text{cis}\,0 = 1$ $\therefore$ roots are $1, p, p^2, p^3$ and p^4			

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No response; no relevant evidence.	1 partial solution.	1u	2u	3u	1r	2r	1t with 1 minor error.	1 t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 12	13 – 18	19 – 24