Assessment Schedule – 2016

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

Q1	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	s clearly marked at (1, 2)	Correct solution.		
(b)	A = –6	Correct solution.		
(c)	$\left(5-\sqrt{x}\right)^2 = x-p$ $25-10\sqrt{x}+x = x-p$ $25-10\sqrt{x} = -p$ $10\sqrt{x} = p+25$ $100x = (p+25)^2$ $x = \dfrac{(p+25)^2}{100}$	Correct 3rd line.	Correct solution with correct, logical working.	
(d)	$(1+2\mathrm{i})^2 + \dfrac{1+2\mathrm{i}}{1-2\mathrm{i}}$ $= -3+4\mathrm{i} + \dfrac{(1+2\mathrm{i})(1+2\mathrm{i})}{(1-2\mathrm{i})(1+2\mathrm{i})}$ $= -3+4\mathrm{i} + \dfrac{-3+4\mathrm{i}}{5}$ $= -3\frac{3}{5} + 4\frac{4}{5}\mathrm{i}$	Correct calculation of w^2 or $\dfrac{w}{\bar{w}}$	Correct solution with working clearly shown.	
(e)	$\lvert x+y\mathrm{i}-2+3\mathrm{i}\rvert = \lvert x+y\mathrm{i}-1\rvert$ $\lvert x-2+(y+3)\mathrm{i}\rvert = \lvert x-1+\mathrm{i}y\rvert$ $\sqrt{(x-2)^2+(y+3)^2} = \sqrt{(x-1)^2+y^2}$ $(x-2)^2+(y+3)^2 = (x-1)^2+y^2$ $x^2-4x+4+y^2+6y+9 = x^2-2x+1+y^2$ $-4x+13+6y = -2x+1$ $6y = 2x-12$ $y = \frac{1}{3}x-2$ $\therefore$ Gradient = $\frac{1}{3}$		Correct 4th line.	Correct gradient with correct working.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial Solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q2	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$x = 3 \pm \sqrt{3}\mathrm{i}$	Correct solution.		
(b)	$(2+3\mathrm{i})(5+m\mathrm{i})$ $= 10 + 2m\mathrm{i} + 15\mathrm{i} + 3m\mathrm{i}^2$ $= 10 - 3m + (15+2m)\mathrm{i}$ $= 22 + 7\mathrm{i}$ $\therefore m = -4$	Correct solution.		
(c)	$z^3 = 8k^6 \mathrm{cis}(\pi)$ $z_1 = 2k^2 \mathrm{cis}\left(\frac{\pi}{3}\right)$ $z_2 = 2k^2 \mathrm{cis}(\pi)$ $z_3 = 2k^2 \mathrm{cis}\left(\frac{5\pi}{3}\right)$ OR $z_3 = 2k^2 \mathrm{cis}\left(\frac{-\pi}{3}\right)$	1 correct solution.	3 correct solutions.	
(d)	$\left\|\frac{4+2\mathrm{i}}{1+\mathrm{i}} \times \frac{1-\mathrm{i}}{1-\mathrm{i}}\right\| = \left\|\frac{4-4\mathrm{i}+2\mathrm{i}-2\mathrm{i}^2}{1-\mathrm{i}^2}\right\|$ $= \left\|\frac{6-2\mathrm{i}}{2}\right\|$ $= \|3-\mathrm{i}\|$ $= \sqrt{3^2+(-1)^2}$ $= \sqrt{10}$	6 – 2i in numerator.	Correct solution with working clearly shown.	
(e)	$2\sqrt{2x+k} = x - 8$ $4(2x+k) = x^2 - 16x + 64$ $8x + 4k = x^2 - 16x + 64$ $x^2 - 24x + 64 - 4k = 0$ One solution $\Rightarrow 24^2 - 4 \times 1 \times (64 - 4k) = 0$ $576 - 256 + 16k = 0$ $320 + 16k = 0$ $k = -20$		Correct equation without $\sqrt{x}$.	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial Solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q3	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$10-5\sqrt{3}$	Correct solution.		
(b)	$\frac{2}{3}\text{cis}\frac{\pi}{12}$	Correct exact solution.		
(c)	$z=3-4\text{i}$ $z-3=4\text{i}$ $z^2-6z+9=-16$ $z^2-6z+25=0$ Other factor is $(z-2)$ $(z-2)(z^2-6z+25)=z^3-8z^2+37z-50$ $\therefore \text{B}=37$	Finding $(z,-2)$ is a factor.	Correct solution.	
(d)	Let $u=a+b\text{i}$ and $v=c+d\text{i}$ $uv=ac+ad\text{i}+bc\text{i}+db\text{i}^2$ $=(ac-bd)+(ad+bc)\text{i}$ $\overline{uv}=(ac-bd)-(ad+bc)\text{i}$ $\overline{u}=a-b\text{i}$ $\overline{v}=c-d\text{i}$ $\overline{u}\cdot\overline{v}=(a-b\text{i})(c-d\text{i})$ $=ac-ad\text{i}-bc\text{i}+bd\text{i}^2$ $=ac-bd-(ad+bc)\text{i}=\overline{uv}$	Correct expression for LHS or RHS.	Correct solution.	
(e)	Let $u=a+b\text{i}$ and $v=c+d\text{i}$ $\lvert u+v\rvert^2=(a+c)^2+(b+d)^2$ $=a^2+2ac+c^2+b^2+2bd+d^2$ $\lvert u-v\rvert^2=(a-c)^2+(b-d)^2$ $=a^2-2ac+c^2+b^2-2bd+d^2$ $\lvert u+v\rvert^2=\lvert u-v\rvert^2$ $\Rightarrow 2ac+2bd=-2ac-2bd$ $4ac+4bd=0$ $ac+bd=0$ $u\overline{v}=(a+b\text{i})(c-di)\text{i}$ $=ac-ad\text{i}+bc\text{i}-bd\text{i}^2$ $=(ac+bd)+(bc-ad)\text{i}$ But from above $ac+bd=0$ $\therefore u\overline{v}$ is purely imaginary		Correct expressions for $\lvert u+v\rvert^2$ and $\lvert u-v\rvert^2$ Expanded.	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial Solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0–8	9–14	15–20	21–24