Assessment Schedule – 2018

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

Q1	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	15	Correct solution.		
(b)	$m^4\text{cis}\dfrac{11\pi}{15}$	Correct solution.		
(c)	$4+4\sqrt{x}+x=x+k$ $4\sqrt{x}=k-4$ $\sqrt{x}=\dfrac{k-4}{4}$ $x=\left(\dfrac{k-4}{4}\right)^2$		Correct solution.	
(d)	$k(1+x^2)=3-8x-x^2$ $k+kx^2-3+8x+x^2=0$ $(k+1)x^2+8x+k-3=0$ For one repeated solution $\Delta=64-4(k+1)(k-3)=0$ $16-(k+1)(k-3)=0$ $16-(k^2-2k-3)=0$ $-k^2+2k+19=0$ $k^2-2k-19=0$ $(k-1)^2-20=0$ or $k=\dfrac{2\pm\sqrt{4+76}}{2}$ $k=1\pm\sqrt{20}$ OR $k=1\pm2\sqrt{5}$	Correct expression for Δ.	Correct exact solutions.	
(e)	$\dfrac{z}{\overline{z}}=\dfrac{(a+b\text{i})}{(a-b\text{i})}\times\dfrac{(a+b\text{i})}{(a+b\text{i})}$ $=\dfrac{a^2+2ab\text{i}-b^2}{a^2+b^2}$ $=\dfrac{a^2-b^2}{a^2+b^2}+\dfrac{2ab}{a^2+b^2}\text{i}$ $c^2+d^2=\left(\dfrac{a^2-b^2}{a^2+b^2}\right)^2+\left(\dfrac{2ab}{a^2+b^2}\right)^2$ $=\dfrac{a^4-2a^2b^2+b^4}{a^4+2a^2b^2+b^4}+\dfrac{4a^2b^2}{a^4+2a^2b^2+b^4}$ $=\dfrac{a^4+2a^2b^2+b^4}{a^4+2a^2b^2+b^4}$ $=1$	Correct expression for $\dfrac{z}{\overline{z}}$ with common denominator (2nd line).	Correct expression for c^2+d^2 found (4th line).	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q 2	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$w = 2 + \mathrm{i}$ clearly shown on Argand diagram	Correct solution.		
(b)	$9 + 3\sqrt{7}$	Correct solution.		
(c)	$z_1 = 3 + \mathrm{i},\ z_2 = 3 - \mathrm{i}$ $(z - 3 - \mathrm{i})(z - 3 + \mathrm{i}) = z^2 - 6z + 10$ $\therefore$ third factor is $(z-4)$, so $z_3 = 4$ $(z-4)(z^2 - 6z + 10) = z^3 - 10z^2 + 34z - 40$ $\therefore \mathrm{A} = -10$	Correct values for z_2 and z_3 OR Correct value of A.	Correct values for z_2 and z_3 AND Correct value of A.	
(d)	$z = \frac{15}{(1-2\mathrm{i})} \times \frac{(1+2\mathrm{i})}{(1+2\mathrm{i})} - 2\mathrm{i}$ $= \frac{15(1+2\mathrm{i})}{5} - 2\mathrm{i}$ $= 3 + 6\mathrm{i} - 2\mathrm{i}$ $= 3 + 4\mathrm{i}$ $\mathrm{mod}(z) = 5$	Correct expression with real denominator.	Correct solution.	
(e)	$\lvert x + y\mathrm{i} - 8 \rvert = \lvert x + \mathrm{i}y - 4 + 2\mathrm{i} \rvert$ $\lvert (x-8) + y\mathrm{i} \rvert = \lvert (x-4) + (2+y)\mathrm{i} \rvert$ $\sqrt{(x-8)^2 + y^2} = \sqrt{(x-4)^2 + (2+y)^2}$ $x^2 - 16x + 64 + y^2 = x^2 - 8x + 16 + y^2 + 4y + 4$ $-16x + 8x + 64 - 20 = 4y$ $-8x + 44 = 4y$ $y = -2x + 11$ $x = 3,\ y = 5$ $\therefore m = 5$	Equation equating moduli without absolute value signs (3rd line).	Equation for locus with squared terms cancelled (line 5).	Correct solution set out logically and clearly.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q 3	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$uv = (3-2\mathrm{i})(2+b\mathrm{i})$ $= 6+3b\mathrm{i}-4\mathrm{i}+2b$ $= (6+2b)+(3b-4)\mathrm{i}$ $b = 4$	Correct solution.		
(b)	$(x-3p)^2 - 5p^2 = 0$ $(x-3p)^2 = 5p^2$ $(x-3p) = \pm\sqrt{5}p$ $x = 3p \pm \sqrt{5}p$ Or by quadratic formula: $x = \dfrac{6p \pm \sqrt{36p^2 - 4\times1\times4p^2}}{2}$ $= \dfrac{6p \pm \sqrt{20p^2}}{2}$ $= 3p \pm \sqrt{5}p$	Correct solution.		
(c)	$z^3 = k^6 \mathrm{cis}\left(\dfrac{-\pi}{2}\right)$ $z_1 = k^2 \mathrm{cis}\left(\dfrac{-\pi}{6}\right)$ $z_2 = k^2 \mathrm{cis}\left(\dfrac{\pi}{2}\right)$ $z_3 = k^2 \mathrm{cis}\left(\dfrac{-5\pi}{6}\right)$	One correct solution (or general solution). Allow equivalent arguments.	All 3 solutions correct. Allow equivalent arguments.	
(d)	$\mathrm{Arg}(w) = \dfrac{\pi}{4} \Rightarrow w = x + x\mathrm{i}$ $w \cdot \overline{w} = (x+x\mathrm{i})(x-x\mathrm{i})$ $= x^2 - x^2\mathrm{i} + x^2\mathrm{i} - x^2\mathrm{i}^2$ $= 2x^2$ $\lvert w \cdot \overline{w} \rvert = \sqrt{(2x^2)^2}$ $= 2x^2$ $2x^2 = 20$ $x^2 = 10$ $x = \sqrt{10}$ $w = \sqrt{10} + \sqrt{10}\mathrm{i}$ Accept $w = 3.16 + 3.16\mathrm{i}$ or equivalent decimal approximations.	Correct expression for $w \cdot \overline{w}$	Correct solution.	

(e)	$\frac{\sqrt{x+k}+\sqrt{x-k}}{\sqrt{x+k}-\sqrt{x-k}}=4$ $\frac{\left(\sqrt{x+k}+\sqrt{x-k}\right)}{\left(\sqrt{x+k}-\sqrt{x-k}\right)}\times\frac{\left(\sqrt{x+k}+\sqrt{x-k}\right)}{\left(\sqrt{x+k}+\sqrt{x-k}\right)}=4$ $\frac{x+k+2\sqrt{x^2-k^2}+x-k}{x+k-(x-k)}=4$ $\frac{2x+2\sqrt{x^2-k^2}}{2k}=4$ $\frac{x+\sqrt{x^2-k^2}}{k}=4$ $x+\sqrt{x^2-k^2}=4k$ $\sqrt{x^2-k^2}=4k-x$ $x^2-k^2=16k^2-8kx+x^2$ $-k^2=16k^2-8kx$ $8kx=17k^2$ $x=\frac{17k}{8}$	2nd line.	A correct expression with simplified rational denominator. (line 4)	Correct solution presented in a logical manner.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 13	14 – 20	21 – 24