Assessment Schedule – 2019

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

Q1	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$x = 2 \pm \sqrt{3}\,\mathrm{i}$	Correct solution.		
(b)	$2(3)^3 - (3)^2 - 4(3) + p = 38$ $54 - 9 - 12 + p = 38$ $p = 5$	Correct solution.		
(c)	$\left\|\frac{u}{v}\right\| = 13$ $\left\|\frac{q+2\mathrm{i}}{1-2\mathrm{i}}\right\| = 13$ $\frac{\sqrt{q^2+4}}{\sqrt{5}} = 13$ $\frac{q^2+4}{5} = 169$ $q^2 + 4 = 845$ $q^2 = 841$ $q = \pm 29$	Third line.	Correct solution.	
(d)	$z = 1 - 2\mathrm{i}$ is one solution $\Rightarrow z = 1 + 2\mathrm{i}$ is also a solution. $\left(z-(1-2\mathrm{i})\right)\left(z-(1+2\mathrm{i})\right)$ $= (z-1+2\mathrm{i})(z-1-2\mathrm{i})$ $= z^2 - z - 2z\mathrm{i} - z + 1 + 2\mathrm{i} + 2\mathrm{i}z - 2\mathrm{i} - 4\mathrm{i}^2$ $= z^2 - 2z + 5$ $\therefore (z^2 - 2z + 5)(az + b) = 2z^3 - 5z^2 + cz - 5$ $a = 2, b = -1$ $\therefore (z^2 - 2z + 5)(2z - 1) = 0$ Other solution is $z = \frac{1}{2}$ $2z^3 - 5z^2 + 12z - 5 = 0$ $c = 12$	Other two solutions found OR *c* found.	Other two solutions found AND *c* found.	

(e)	$\frac{1}{x+\mathrm{i}y}-\frac{1}{1+\mathrm{i}}=1-2\mathrm{i}$ $\frac{1}{x+\mathrm{i}y}-\frac{1(1-\mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})}=1-2\mathrm{i}$ $\frac{1}{x+\mathrm{i}y}-\frac{1-\mathrm{i}}{2}=1-2\mathrm{i}$ $\frac{2}{x+\mathrm{i}y}-1+\mathrm{i}=2-4\mathrm{i}$ $\frac{2}{x+\mathrm{i}y}=3-5\mathrm{i}$ $\frac{x+\mathrm{i}y}{2}=\frac{1}{3-5\mathrm{i}}$ $\frac{x+iy}{2}=\frac{(3+5\mathrm{i})}{(3-5\mathrm{i})(3+5\mathrm{i})}$ $\frac{x+\mathrm{i}y}{2}=\frac{3+5\mathrm{i}}{34}$ $x+\mathrm{i}y=\frac{3+5\mathrm{i}}{17}$ $x=\frac{3}{17}\quad y=\frac{5}{17}$	Correctly rationalises a term.	Correct reduction of expression to two terms. OR Equates real and imaginary parts.	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q 2	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\overline{p}=3+\mathrm{i}$ $\overline{p}-3q=3+\mathrm{i}-3(-2+5\mathrm{i})$ $=3+\mathrm{i}+6-15\mathrm{i}$ $=9-14\mathrm{i}$	Correct solution.		
(b)	$\frac{3}{4-\sqrt{5}}=\frac{3}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}$ $=\frac{12+3\sqrt{5}}{16-5}$ $=\frac{12}{11}+\frac{3\sqrt{5}}{11}$	Correct solution.		
(c)	$z^4=-16p^2\mathrm{i}$ $=16p^2\mathrm{cis}\left(\frac{-\pi}{2}\right)$ $z=\left(16p^2\mathrm{cis}\left(\frac{-\pi}{2}\right)\right)^{\frac{1}{4}}$ $=2\sqrt{p}\,\mathrm{cis}\left(\frac{-\pi}{8}\right)$ Solutions are $\frac{\pi}{2}$ apart. $z_1=2\sqrt{p}\,\mathrm{cis}\left(\frac{-\pi}{8}\right)$ $z_2=2\sqrt{p}\,\mathrm{cis}\left(\frac{3\pi}{8}\right)$ $z_3=2\sqrt{p}\,\mathrm{cis}\left(\frac{7\pi}{8}\right)$ $z_4=2\sqrt{p}\,\mathrm{cis}\left(\frac{-5\pi}{8}\right)$ or equivalent.	One correct solution, or all arguments correct.	Four correct solutions.	
(d)	$z=\frac{(\sqrt{3}+m\mathrm{i})(1-\sqrt{3}\mathrm{i})}{(1+\sqrt{3}\mathrm{i})(1-\sqrt{3}\mathrm{i})}$ $=\frac{\sqrt{3}-3\mathrm{i}+m\mathrm{i}-\sqrt{3}m\mathrm{i}^2}{4}$ $=\frac{\sqrt{3}+\sqrt{3}m+(m-3)\mathrm{i}}{4}$ z is purely real $\Rightarrow m=3$	Correct expanded expression (without i^2).	Correct solution.	

(e)	$z = x + \mathrm{i}y$ $\lvert z \rvert = 1 \Rightarrow x^2 + y^2 = 1$ $\dfrac{1+z}{1-z} = \dfrac{1+(x+\mathrm{i}y)}{1-(x+\mathrm{i}y)}$ $= \dfrac{(1+x)+\mathrm{i}y}{(1-x)-\mathrm{i}y}$ $= \dfrac{(1+x)+\mathrm{i}y}{(1-x)-\mathrm{i}y} \times \dfrac{(1-x)+\mathrm{i}y}{(1-x)+\mathrm{i}y}$ $= \dfrac{(1+x)(1-x)+(1+x)\mathrm{i}y + iy(1-x)+\mathrm{i}^2y^2}{(1-x)^2 - \mathrm{i}^2y^2}$ $= \dfrac{1-x^2+\mathrm{i}y+\mathrm{i}yx+\mathrm{i}y-\mathrm{i}yx-y^2}{(1-x)^2+y^2}$ $= \dfrac{1-(x^2+y^2)+2\mathrm{i}y}{(1-x)^2+y^2}$ $= \dfrac{2\mathrm{i}y}{(1-x)^2+y^2}$ which is purely imaginary.	Real and imaginary part grouped (4th line)	Correct expansion (without i^2).	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q 3	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\frac{u}{v}=\frac{q^2\text{cis}\left(\frac{3\pi}{4}\right)}{q^3\text{cis}\left(\frac{\pi}{3}\right)}$ $=\frac{1}{q}\text{cis}\left(\frac{3\pi}{4}-\frac{\pi}{3}\right)$ $=\frac{1}{q}\text{cis}\left(\frac{5\pi}{12}\right)$	Correct solution.		
(b)	$x = 1$ and $y = -1$	Correct solution.		
(c)	$2\sqrt{x-3}=w\sqrt{x}$ $4(x-3)=w^2x$ $4x-12=w^2x$ $4x-w^2x=12$ $x(4-w^2)=12$ $x=\frac{12}{(4-w^2)}$	Correct equation with terms in x gathered (line 4).	Correct solution.	
(d)	$\frac{u}{v}=\frac{1+p\text{i}}{5+3\text{i}}\times\frac{5-3\text{i}}{5-3\text{i}}$ $=\frac{5-3\text{i}+5p\text{i}-3p\text{i}^2}{34}$ $=\frac{5+3p}{34}+\frac{5p-3}{34}\text{i}$ $\arg\left(\frac{u}{v}\right)=\left(\frac{\pi}{4}\right)$ $\Rightarrow \text{Re}\left(\frac{u}{v}\right)=\text{Im}\left(\frac{u}{v}\right)$ $\Rightarrow 5+3p=5p-3$ $8=2p$ $p=4$	3rd line or equivalent with real and imaginary terms grouped.	Correct solution.	

(e)	$x^2+3kx+k^2=7x+3k$ $x^2+(3k-7)x+(k^2-3k)=0$ $b^2-4ac=(3k-7)^2-4\times1\times(k^2-3k)$ $=9k^2-42k+49-4k^2+12k$ $=5k^2-30k+49$ $=5(k^2-6k)+49$ $=5\left[(k-3)^2-9\right]+49$ $=5(k-3)^2+4$ $5(k-3)^2+4>0$ for all values of k. $\therefore$ Quadratic equation will always have 2 real solutions.	Correct expression for b^2-4ac.	Complete the square correctly started (line 7) or equivalent.	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 13	14 – 19	20 – 24