Assessment Schedule – 2020

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$st = (2+3\mathrm{i})(3+k\mathrm{i})$ $= 6+2k\mathrm{i}+9\mathrm{i}+3k\mathrm{i}^2$ $= (6-3k)+(2k+9)\mathrm{i} = 21-\mathrm{i}$ $k = -5$	Correct solution.		
(b)	$x^2+4rx+r=0$ $(4r)^2-4\times1\times r=0$ $16r^2-4r=0$ $4r(4r-1)=0$ $r=0$ or $r=\frac{1}{4}$	Correct solution.		
(c)	$2\sqrt{x}-5=\sqrt{4x-g}$ $4x-20\sqrt{x}+25=4x-g$ $25+g=20\sqrt{x}$ $x=\left(\frac{25+g}{20}\right)^2$	Correct squaring of both sides.	Correct solution.	
(d)	$\frac{k+k\mathrm{i}}{1-\mathrm{i}}+\frac{2k}{1+\mathrm{i}}=\frac{(k+k\mathrm{i})(1+\mathrm{i})+2k(1-\mathrm{i})}{(1-\mathrm{i})(1+\mathrm{i})}$ $=\frac{k+k\mathrm{i}+k\mathrm{i}+k\mathrm{i}^2+2k-2k\mathrm{i}}{2}$ $=\frac{2k}{2}=k$	Correct expansion with common denominator (line 2).	Correct solution.	

(e)	$\frac{1+T^2}{2T}=\frac{1+\left(\frac{a-bi}{a+bi}\right)^2}{2\left(\frac{a-bi}{a+bi}\right)}$ $=\frac{\left(\frac{a+bi}{a+bi}\right)^2+\left(\frac{a-bi}{a+bi}\right)^2}{\frac{2(a-bi)}{(a+bi)}}$ $=\frac{(a+bi)^2+(a-bi)^2}{(a+bi)^2}\times\frac{(a+bi)}{2(a-bi)}$ $=\frac{(a+bi)^2+(a-bi)^2}{(a+bi)\times 2(a-bi)}$ $=\frac{a^2+2abi+b^2i^2+a^2-2abi+b^2i^2}{2\left(a^2+abi-abi-b^2i^2\right)}$ $=\frac{2\left(a^2-b^2\right)}{2\left(a^2+b^2\right)}$ $=\frac{\left(a^2-b^2\right)}{\left(a^2+b^2\right)}$	This line, or equivalent.	This line, or equivalent.	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$2\times 2^3 + q\times 2^2 - 17\times 2 - 10 = 0$ $16 + 4q - 34 - 10 = 0$ $q = 7$	Correct solution.		
(b)	$\lvert 5+3k\mathrm{i}\rvert = 13$ $25 + 9k^2 = 169$ $9k^2 = 144$ $k^2 = 16$ $k = \pm 4$	Correct solutions (both required). Can be done by inspection.		
(c)	$2z^3 - 15z^2 + bz - 30 = 0$ $z = 3+\mathrm{i}$ a solution $\Rightarrow z = 3-\mathrm{i}$ a solution $(z-3-\mathrm{i})(z-3+\mathrm{i}) = z^2 - 6z + 10$ $2z^3 - 15z^2 + bz - 30 = (2z-3)(z^2-6z+10)$ Other solution is $z = \dfrac{3}{2}$ and $b = 38$.	Other TWO solutions found. OR b found.	Other TWO solutions found. AND b found.	
(d)	$u = p + p\mathrm{i} \quad v = -q + q\mathrm{i}$ $\dfrac{u}{v} = \dfrac{p+p\mathrm{i}}{-q+q\mathrm{i}} \times \dfrac{-q-q\mathrm{i}}{-q-q\mathrm{i}}$ $= \dfrac{-pq - pq\mathrm{i} - pq\mathrm{i} - pq\mathrm{i}^2}{2q^2}$ $= \dfrac{-2pq\mathrm{i}}{2q^2} = \dfrac{-p\mathrm{i}}{q}$ $\arg\left(\dfrac{u}{v}\right) = \dfrac{-\pi}{2}$ OR $\arg(u) = \dfrac{\pi}{4} \quad \arg(v) = \dfrac{3\pi}{4}$ $\arg\left(\dfrac{u}{v}\right) = \dfrac{\pi}{4} - \dfrac{3\pi}{4} = \dfrac{-\pi}{2}$	3rd line with i^2 substituted. OR Correct $\arg(u)$ and $\arg(v)$.	Correct solution. Accept other correct expressions of argument.	
(e)	$\lvert z+\mathrm{i}\rvert^2 + \lvert z-\mathrm{i}\rvert^2 = 10 \quad$ Let $z = x + y\mathrm{i}$ $\lvert x+y\mathrm{i}+\mathrm{i}\rvert^2 + \lvert x+y\mathrm{i}-\mathrm{i}\rvert^2 = 10$ $x^2 + (y+1)^2 + x^2 + (y-1)^2 = 10$ $2x^2 + y^2 + 2y + 1 + y^2 - 2y + 1 = 10$ $2x^2 + 2y^2 = 8$ $x^2 + y^2 = 4$		Correct expanded expression (line 4).	Correct solution.

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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$6k^2\text{cis}\left(\frac{2\pi}{3}\right)$	Correct solution.		
(b)	$z=5-\text{i} \quad w=-2+3\text{i}$ $\lvert z\rvert^2=26 \quad \lvert w\rvert^2=13$ $\therefore \lvert z\rvert^2=2\lvert w\rvert^2$	Correct solution.		
(c)	$\frac{z\bar{z}}{z+\bar{z}}=\frac{(a+b\text{i})(a-b\text{i})}{a+b\text{i}+a-b\text{i}}$ $=\frac{a^2+b^2}{2a}$ which is real	Both numerator and denominator evaluated correctly.	Correct solution with a statement indicating real part only OR imaginary part = 0.	
(d)	$z^4=-16k^8=16k^8\text{cis}(\pi)$ $z_1=2k^2\text{cis}\left(\frac{\pi}{4}\right)$ $z_2=2k^2\text{cis}\left(\frac{3\pi}{4}\right)$ $z_3=2k^2\text{cis}\left(\frac{-3\pi}{4}\right)$ $z_4=2k^2\text{cis}\left(\frac{-\pi}{4}\right)$	z^4 written correctly in polar form and one correct solution, OR all arguments correct.	Four correct solutions. Accept equivalents in degrees.	

(e)	$\|u+v\|=\|u-v\|$ $u+v=a+bi+c+di$ $=a+c+(b+d)i$ $u-v=a+bi-c-di$ $=a-c+(b-d)i$ $\|u+v\|=\|u-v\|$ $\Rightarrow (a+c)^2+(b+d)^2=(a-c)^2+(b-d)^2$ $a^2+2ac+c^2+b^2+2bd+d^2$ $=a^2-2ac+c^2+b^2-2bd+d^2$ $2ac+2bd=-2ac-2bd$ $4ac+4bd=0$ $ac+bd=0$ $\frac{u}{v}=\frac{(a+bi)}{(c+di)}\times\frac{(c-di)}{(c-di)}$ $=\frac{ac-adi+bci-bdi^2}{c^2+d^2}$ $=\frac{ac+bd+(bc-ad)i}{c^2+d^2}$ But $ac+bd=0$ $\therefore \frac{u}{v}=\frac{(bc-ad)i}{c^2+d^2}$ which is purely imaginary	Correct expression for $\|u+v\|=\|u-v\|$ (Line beginning with arrow.)	Correct simplified expression for $\|u+v\|=\|u-v\|$ $ac+bd=0$ or equivalent.	Correct solution.

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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 8	9 – 14	15 – 20	21 – 24