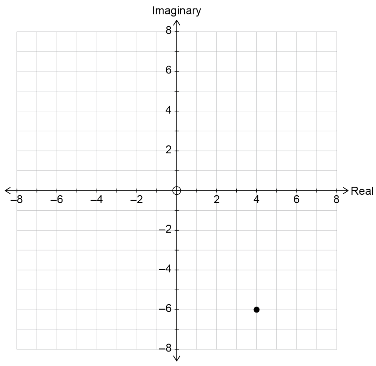
Assessment Schedule – 2021

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$(d+5\mathrm{i})(3-4\mathrm{i})=38-9\mathrm{i}$ $3d-4d\mathrm{i}+15\mathrm{i}-20\mathrm{i}^2=38-9\mathrm{i}$ $3d+20+(15-4d)\mathrm{i}=38-9\mathrm{i}$ $d=6$	Correct solution.		
(b)	$z=\dfrac{26(2-3\mathrm{i})}{(2+3\mathrm{i})(2-3\mathrm{i})}$ $=\dfrac{26(2-3\mathrm{i})}{13}$ $=4-6\mathrm{i}$  	Correct solution correctly plotted on Argand diagram.		
(c)	$f(-1)=f(2)$ $-1+3-a+b=8+12+2a+b$ $2-20=2a+a$ $a=-6$ $f(-2)=0$ $-8+12-2a+b=0$ $16+b=0$ $b=-16$	Correct value for *a* OR *b*.	Correct values for *a* AND *b*.	
(d)	$\arg\left(\dfrac{1+3\mathrm{i}-1}{1+3\mathrm{i}-2\mathrm{i}}\right)=\arg\left(\dfrac{3\mathrm{i}}{1+\mathrm{i}}\right)$ $=\arg\left(\dfrac{3\mathrm{i}(1-\mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})}\right)$ $=\arg\left(\dfrac{3\mathrm{i}-3\mathrm{i}^2}{1-\mathrm{i}^2}\right)$ $=\arg\left(\dfrac{3+3\mathrm{i}}{2}\right)$ $=\arg\left(\dfrac{3}{2}+\dfrac{3i}{2}\right)$ $=\dfrac{\pi}{4}$	Correct simplification to $\left(\dfrac{3\mathrm{i}}{1+\mathrm{i}}\right)$.	Correct solution.	

(e)	$\frac{z-2\text{i}}{z-4}=\frac{x+y\text{i}-2\text{i}}{x+y\text{i}-4}$ $=\frac{x+(y-2)\text{i}}{(x-4)+y\text{i}}$ $=\frac{(x+(y-2)\text{i})((x-4)-y\text{i})}{((x-4)+y\text{i})((x-4)-y\text{i})}$ $=\frac{x(x-4)-xy\text{i}+(y-2)(x-4)\text{i}-y(y-2)\text{i}^2}{(x-4)^2-y^2\text{i}^2}$ $=\frac{x(x-4)-xy\text{i}+(y-2)(x-4)\text{i}+y(y-2)}{(x-4)^2+y^2}$ Real part $=0$ $\Rightarrow x(x-4)+y(y-2)=0$ $x^2-4x+y^2-2y=0$ $(x-2)^2-4+(y-1)^2-1=0$ $(x-2)^2+(y-1)^2=5$	Line 3.	Line 5.	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$u = 2\text{cis}\left(\frac{\pi}{2}\right)$ $z = \frac{u}{w} = \frac{2\text{cis}\frac{\pi}{2}}{2\text{cis}\frac{2\pi}{3}}$ $= \text{cis}\left(\frac{\pi}{2} - \frac{2\pi}{3}\right)$ $= \text{cis}\left(-\frac{\pi}{6}\right)$ $= \cos\left(-\frac{\pi}{6}\right) + \text{i}\sin\left(-\frac{\pi}{6}\right)$ $= \frac{\sqrt{3}}{2} - 0.5\text{i}$ or $0.866 - 0.5\text{i}$	Correct solution. Accept answer in rectangular or polar form.		
(b)	$x^2 - 12qx + 20q^2 = 0$ $(x-6q)^2 - 36q^2 + 20q^2 = 0$ $(x-6q)^2 = 16q^2$ $x - 6q = \pm 4q$ $x = 6q \pm 4q$ $x = 10q$ or $2q$	Correct solution.		
(c)	$\frac{(a+b\text{i})(b+a\text{i})}{(b-a\text{i})(b+a\text{i})}$ $= \frac{ab + a^2\text{i} + b^2\text{i} + ab\text{i}^2}{b^2 + ab\text{i} - ab\text{i} - a^2\text{i}^2}$ $= \frac{ab + (a^2+b^2)\text{i} - ab}{b^2 + a^2}$ $= \frac{(a^2+b^2)\text{i}}{b^2+a^2}$ $= \text{i}$	Correct 2nd line.	Correct solution.	
(d)	$z^3 = k^6 + k^6\text{i}$ $= \sqrt{2}\,k^6\text{cis}\left(\frac{\pi}{4}\right)$ $z_1 = \left(\sqrt{2}\right)^{\frac{1}{3}} k^2\text{cis}\left(\frac{\pi}{12}\right) = 2^{\frac{1}{6}} k^2\text{cis}\left(\frac{\pi}{12}\right)$ $z_2 = \left(\sqrt{2}\right)^{\frac{1}{3}} k^2\text{cis}\left(\frac{3\pi}{4}\right) = 2^{\frac{1}{6}} k^2\text{cis}\left(\frac{3\pi}{4}\right)$ $z_3 = \left(\sqrt{2}\right)^{\frac{1}{3}} k^2\text{cis}\left(\frac{-7\pi}{12}\right) = 2^{\frac{1}{6}} k^2\text{cis}\left(\frac{-7\pi}{12}\right)$	One correct solution. $2^{\frac{1}{6}} = 1.122$	Three correct solutions. Accept $\frac{17\pi}{12}$.	

(e)	$\lvert x+\mathrm{i}y+16\rvert = 4\lvert x+\mathrm{i}y+1\rvert$ $\lvert(x+16)+\mathrm{i}y\rvert = 4\lvert(x+1)+\mathrm{i}y\rvert$ $\sqrt{x^2+32x+256+y^2} = 4\sqrt{x^2+2x+1+y^2}$ $x^2+32x+256+y^2 = 16\left(x^2+2x+1+y^2\right)$ $x^2+32x+256+y^2 = 16x^2+32x+16+16y^2$ $240 = 15x^2+15y^2$ $16 = x^2+y^2$ $4 = \sqrt{x^2+y^2}$ $\therefore \lvert z\rvert = 4$	3rd line.	5th line.	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\sqrt{11}$	Correct solution.		
(b)	$\frac{18}{\left(4-2\sqrt{3}\right)}\times\frac{\left(4+2\sqrt{3}\right)}{\left(4+2\sqrt{3}\right)}=\frac{72+36\sqrt{3}}{16-12}$ $=18+9\sqrt{3}$	Correct solution.		
(c)	$z_1=2+5\text{i}$ $z_2=2-5\text{i}$ $\left(z-2-5\text{i}\right)\left(z-2+5\text{i}\right)=z^2-2z+5z\text{i}-2z+4-10\text{i}-5\text{i}z+10\text{i}-25\text{i}^2$ $=z^2-4z+4+25$ $=z^2-4z+29$ $f\left(z\right)=\left(z^2-4z+29\right)\left(pz+q\right)$ $p=4$ Coeff of z^2: $\left(z^2-4z+29\right)\left(4z+q\right)=-19$ $q-16=-19$ $q=-3$ $f\left(z\right)=\left(z^2-4z+29\right)\left(4z-3\right)$ $z_3=\frac{3}{4}$ $A=-87$	The other two solutions found. OR A found.	The other two solutions found. AND A found.	
(d)	$6\sqrt{2x}-5=6\sqrt{2x+m}$ $\left(6\sqrt{2x}-5\right)^2=\left(6\sqrt{2x+m}\right)^2$ $36\times2x-60\sqrt{2x}+25=36\left(2x+m\right)$ $72x-60\sqrt{2x}+25=72x+36m$ $-60\sqrt{2x}=36m-25$ $\sqrt{2x}=\frac{25-36m}{60}$ $2x=\left(\frac{25-36m}{60}\right)^2$ $x=\frac{\left(25-36m\right)^2}{7200}$	Correct 3rd line.	Correct solution.	

(e)	$z^2 = \mathrm{i}\left(\lvert z\rvert^2 - 4\right)$ $(x+\mathrm{i}y)^2 = \mathrm{i}\left(x^2+y^2-4\right)$ $x^2 + 2xy\mathrm{i} + y^2\mathrm{i}^2 = \left(x^2+y^2-4\right)\mathrm{i}$ $x^2 - y^2 + 2xy\mathrm{i} = \left(x^2+y^2-4\right)\mathrm{i}$ Real component: $x^2 - y^2 = 0$ $x^2 = y^2$ $x = y \text{ or } x = -y$ Imaginary component: $x^2 + y^2 - 4 = 2xy$ *If* $x = y$, then $2x^2 - 4 = 2x^2$ impossible *If* $x = -y$, then $2x^2 - 4 = -2x^2$ $4x^2 = 4$ $x = \pm 1$ $x = 1 \Rightarrow y = -1 \Rightarrow z = 1 - \mathrm{i}$ $x = -1 \Rightarrow y = 1 \Rightarrow z = -1 + \mathrm{i}$	Finding $x^2 = y^2$.	Identifying $x = -y$ is only possibility.	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 14	15 – 19	20 – 24