Assessment Schedule – 2022

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\frac{12k}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{12k-12k\sqrt{5}}{-4}$ $= -3k+3k\sqrt{5}$	Correct solution.		
(b)	$\frac{u}{v} = m^3 \text{cis}\left(\frac{\pi}{3}-\frac{\pi}{5}\right)$ $= m^3 \text{cis}\left(\frac{2\pi}{15}\right)$	Correct solution.		
(c)	$uv = (3+2\text{i})(4+2\text{i})$ $= 8+14\text{i}$ $uvw = (8+14\text{i})(2+k\text{i})$ $= (16-14k)+(8k+28)\text{i}$ $\text{Arg}(uvw) = \frac{\pi}{4}$ means $\text{Re} = \text{Im}$ $(16-14k) = (8k+28)$ $-12 = 22k$ $k = \frac{-12}{22} = \frac{-6}{11}$	$(16 - 14k) + (8k + 28)\text{i}$	Correct solution.	
(d)	$x - 2\sqrt{x+p} = -5$ $x+5 = 2\sqrt{x+p}$ $x^2+10x+25 = 4(x+p)$ $x^2+6x+25-4p = 0$ $b^2-4ac = 0$ $36-4(25-4p) = 0$ $36-100+16p = 0$ $16p = 64$ $p = 4$	Correct quadratic.	Correct solution.	
(e)	$\lvert w+z\rvert^2 - \lvert w-\bar{z}\rvert^2 = 4\,\text{Re}(w)\,\text{Re}(z)$ *Let* $w = a+b\text{i}$ *and* $z = c+d\text{i}$ $w+z = a+b\text{i}+c+d\text{i}$ $= a+c+(b+d)\text{i}$ $w-\bar{z} = a+b\text{i}-(c-d)\text{i}$ $= a-c+(b+d)\text{i}$ $\lvert w+z\rvert^2 - \lvert w-\bar{z}\rvert^2 = (a+c)^2+(b+d)^2-((a-c)^2+(b+d)^2)$ $= a^2+2ac+c^2+b^2+2bd+d^2-(a^2-2ac+c^2+b^2+2bd+d^2)$ $= 4ac$ $= 4\,\text{Re}(w)\,\text{Re}(z)$		Correct expanded form for $\lvert w+z\rvert^2 - \lvert w-\bar{z}\rvert^2$	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$(-2)^3-3(-2)^2-2b+9=3$ $-8-12-2b+9=3$ $-2b=14$ $b=-7$	Correct solution.		
(b)	$Let\ z=a+bi$ $a+bi+4(a-bi)=15+12i$ $5a-3bi=15+12i$ $a=3,\ b=-4$ $z=3-4i$	Correct solution.		
(c)	$z+4\overline{)z^3-2z^2+hz+180}$ quotient $z^2-6z+45$ z^3+4z^2 $-6z^2+hz$ $-6z^2-24z$ $(h+24)z+180$ $45z+180$ 0 $h+24=45$ $h=21$ $z^2-6z+45=0$ $(z-3)^2+36=0$ $(z-3)^2=-36$ $z-3=\pm 6i$ $z=3\pm 6i$	Correct value of *h*. OR Other solutions found correctly.	Correct value of h. AND Other solutions found algebraically.	
(d)	$w=\frac{4}{z}-2 \quad z=1-\sqrt{3}i$ $w=\frac{4}{1-\sqrt{3}i}-2$ $=\frac{4}{1-\sqrt{3}i}\times\frac{1+\sqrt{3}i}{1+\sqrt{3}i}-2$ $=\frac{4+4\sqrt{3}i}{4}-2$ $=1+\sqrt{3}i-2$ $=-1+\sqrt{3}i$ $\arg(w)=\frac{2\pi}{3}$ or $\arg(w)=120°$	Correct expression for *w* with rationalised denominator.	Correct solution.	

(e)	$\lvert z+\mathrm{i}\rvert = 2\lvert z-5\mathrm{i}\rvert$ $\lvert x+\mathrm{i}y+\mathrm{i}\rvert = 2\lvert x+\mathrm{i}y-5\mathrm{i}\rvert$ $\lvert x+(y+1)\mathrm{i}\rvert = 2\lvert x+(y-5)\mathrm{i}\rvert$ $\sqrt{x^2+(y+1)^2} = 2\sqrt{x^2+(y-5)^2}$ $x^2+(y+1)^2 = 4(x^2+(y-5)^2)$ $x^2+y^2+2y+1 = 4x^2+4y^2-40y+100$ $0 = 3x^2+3y^2-42y+99$ $x^2+y^2-14y+33=0$ $x^2+(y-7)^2-16=0$ $x^2+(y-7)^2=16$		Correct expanded quadratic.	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$4+9\text{i}$ correctly plotted.	Correct solution.		
(b)	$z^2+6kz+152k^2=0$ $(z+3k)^2+6k^2=0$ $(z+3k)^2=-6k^2$ $z+3k=\pm\sqrt{6}k\text{i}$ $z=-3k\pm\sqrt{6}k\text{i}$	Correct solution.		
(c)	$z^3=-k^6\text{i}$ $=k^6\text{cis}\left(\frac{-\pi}{2}\right)$ Solution interval $=\frac{2\pi}{3}$ $z_1=k^2\text{cis}\left(\frac{-\pi}{6}\right)$ $z_2=k^2\text{cis}\left(\frac{3\pi}{6}\right)=k^2\text{cis}\left(\frac{\pi}{2}\right)$ $z_3=k^2\text{cis}\left(\frac{7\pi}{6}\right)=k^2\text{cis}\left(\frac{-5\pi}{6}\right)$	One correct solution.	Three correct solutions.	
(d)	$\lvert z\rvert-z=\text{i}$ $z=a+b\text{i}$ $\sqrt{a^2+b^2}-(a+b\text{i})=\text{i}$ $\sqrt{a^2+b^2}-a-b\text{i}=\text{i}$ Equating imaginary parts: $-b\text{i}=\text{i}$ $b=-1$ Equating real parts: $\sqrt{a^2+(-1)^2}-a=0$ $\sqrt{a^2+1}=a$ $a^2+1=a^2$ $1=0$ Not possible.	Correct value of b.	Correct solution.	

(e)	$\frac{i}{z}+\frac{3}{\bar{z}}=1$ $\frac{i}{a+bi}+\frac{3}{a-bi}=1$ $\frac{i(a-bi)+3(a+bi)}{(a+bi)(a-bi)}=1$ $\frac{ai+b+3a+3bi}{a^2+b^2}=1$ $\frac{b+3a}{a^2+b^2}+\frac{(a+3b)i}{a^2+b^2}=1$ Equating imaginary parts: $a+3b=0$ or $a=-3b$ Equating real parts: $\frac{b+3a}{a^2+b^2}=1$ $b+3a=a^2+b^2$ $-8b=10b^2$ $2b(5b+4)=0$ $b=0$ or $b=\frac{-4}{5}$ If $b=0$, then $a=-3b=0$ not allowed $b=\frac{-4}{5}$ $a=-3b=\frac{12}{5}$ $\left(z=\frac{12}{5}-\frac{4}{5}i\right)$		$a=-3b$.	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 14	15 – 20	21 – 24