Assessment Schedule – 2023

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\left(5-2\sqrt{p}\right)\left(5-2\sqrt{p}\right)=25-10\sqrt{p}-10\sqrt{p}+4p$ $=25+4p-20\sqrt{p}$	• Required expression.		
(b)	No real roots so $b^2-4ac<0$ $16-4\times4\times(3r-2)<0$ $16-48r+32<0$ $-48r+48<0$ $-48r<-48$ $r>1$	• Correct inequality.		
(c)	$\frac{z}{w}=\frac{p+q\text{i}}{a+b\text{i}}=\frac{(p+q\text{i})(a-b\text{i})}{(a+b\text{i})(a-b\text{i})}$ $=\frac{ap-bp\text{i}+aq\text{i}-bq\text{i}^2}{a^2-b^2\text{i}^2}$ $=\frac{ap-bp\text{i}+aq\text{i}+bq}{a^2+b^2}$ #(1) $=\frac{ap+bq+(aq-bp)\text{i}}{a^2+b^2}$ $\text{Re}\left(\frac{z}{w}\right)=0\Rightarrow\frac{ap+bq}{a^2+b^2}=0$ $\Rightarrow ap+bq=0$ $\Rightarrow ap=-bq$ As required.	• Reaching stage #(1) .	• Proof completed.	
(d)	$z_1=5-\text{i}$ so $z_2=5+\text{i}$ $z-5=\text{i}\Rightarrow(z-5)^2=\text{i}^2$ $\Rightarrow z^2-10z+26=0$ $f(z)=(Az+B)(z^2-10z+26)$ $\Rightarrow A=1;\ B=2$ i.e. $f(z)=(z+2)(z^2-10z+26)$ So $z_3=-2$ and $d=52$	• The other two solutions found. OR • d found.	• The other two solutions found. AND • d found.	

(e)	$\left\lvert \frac{u}{v}+k \right\rvert = \sqrt{k+2}$ $\left\lvert \frac{3+\mathrm{i}}{1+2\mathrm{i}}+k \right\rvert = \sqrt{k+2}$ $\left\lvert \frac{(3+\mathrm{i})(1-2\mathrm{i})}{(1+2\mathrm{i})(1-2\mathrm{i})}+k \right\rvert = \sqrt{k+2}$ $\left\lvert \frac{5-5\mathrm{i}}{5}+k \right\rvert = \sqrt{k+2}$ $\lvert 1-\mathrm{i}+k \rvert = \sqrt{k+2}$ $\sqrt{(1+k)^2+1} = \sqrt{k+2}$ #(1) $(1+k)^2+1 = k+2$ $k^2+2k+2 = k+2$ $k^2+k=0 \quad k(k+1)=0$ Either $k=0$ or $k=-1$	• Expressing $\frac{u}{v}$ in the form $1-\mathrm{i}$.	• Reaching stage #(1).	E7 Not including $k=0$ as a valid solution. OR Correct solution but with one minor error. E8 Finding $k=0$ and $k=-1$. Both solutions required, with valid and clear justification.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$q^4 \text{cis}\left(\frac{9\pi}{40}\right)$	• Correct answer.		
(b)	$\lvert z-w \rvert = \lvert (1+k\text{i})-(7-k\text{i}) \rvert$ $= \lvert -6+2k\text{i} \rvert$ $= \sqrt{36+4k^2}$	• Correct expression.		
(c)	$13z = (11-3\text{i})(z+1)$ $13z = 11z + 11 - 3\text{i}z - 3\text{i}$ $2z + 3\text{i}z = 11 - 3\text{i}$ $z(2+3\text{i}) = 11 - 3\text{i}$ $z = \frac{11-3\text{i}}{2+3\text{i}}$ #(1) $z = 1 - 3\text{i}$ $\text{Arg}\, z = -71.6°$ (or -1.25 rads) (or $288.4°$)	• Reaching stage #(1).	• Correct value for Arg z.	
(d)	$z^3 = -64m^{12} = 64m^{12}\text{cis}\,\pi$ $\vartheta_1 = \frac{\pi}{3}$ $\vartheta_2 = \pi$ $\vartheta_3 = \frac{5\pi}{3} = -\frac{\pi}{3}$ $z_1 = 4m^4 \text{cis}\frac{\pi}{3}$ $z_2 = 4m^4 \text{cis}\,\pi$ $z_3 = 4m^4 \text{cis}\frac{5\pi}{3} = 4m^4 \text{cis}\left(-\frac{\pi}{3}\right)$ Or equivalent.	• One correct solution.	• All three correct solutions, with appropriate justification.	

(e)	Let $z = x + yi$, then $\lvert x + yi - 2 + i \rvert = \sqrt{3}$ $\lvert (x-2) + (y+1)i \rvert = \sqrt{3}$ $\sqrt{(x-2)^2 + (y+1)^2} = \sqrt{3}$ #(1) $(x-2)^2 + (y+1)^2 = 3$ Let $y = mx - 1$, then $(x-2)^2 + (mx - 1 + 1)^2 = 3$ $x^2 - 4x + 4 + m^2x^2 = 3$ $(1+m^2)x^2 - 4x + 1 = 0$ #(2) Tangent gives $b^2 - 4ac = 0$ $16 - 4(1+m^2) \times 1 = 0$ $16 - 4 - 4m^2 = 0$ $12 = 4m^2$ $m^2 = 3$ $m = \sqrt{3}$ Do not penalise $m = \pm\sqrt{3}$	• Reaching stage #(1).	• Reaching stage #(2) .	E 7 Correct solution but with one minor error. E 8 Correct value Of m with valid and clear justification.

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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$f(-3)=30$ $-54+9p-21-3=30$ $9p-78=30$ $9p=108$ $p=12$	• Correct value of p.		
(b)	$\dfrac{n-\mathrm{i}}{2-3\mathrm{i}}=3+4\mathrm{i}$ $n-\mathrm{i}=(3+4\mathrm{i})(2-3\mathrm{i})$ $n-\mathrm{i}=6-9\mathrm{i}+8\mathrm{i}+12$ $n-\mathrm{i}=18-\mathrm{i}$ $n=18$ Alternative method: $\dfrac{n-\mathrm{i}}{2-3\mathrm{i}}=3+4\mathrm{i}$ $\dfrac{(n-\mathrm{i})(2+3\mathrm{i})}{(2-3\mathrm{i})(2+3\mathrm{i})}=3+4\mathrm{i}$ $\dfrac{2n+3n\mathrm{i}-2\mathrm{i}+3}{4+9}=3+4\mathrm{i}$ $\dfrac{2n+3+(3n-2)\mathrm{i}}{13}=3+4\mathrm{i}$ Comparing real parts gives: $\dfrac{2n+3}{13}=3$, giving $n=18$ OR Comparing imaginary parts gives: $\dfrac{3n-2}{13}=4$, giving $n=18$	• Correct value of n.		
(c)	$16(4x-w)=(5-8\sqrt{x})(5-8\sqrt{x})$ $64x-16w=25-80\sqrt{x}+64x$ #(1) $80\sqrt{x}=25+16w$ $\sqrt{x}=\dfrac{25+16w}{80}$ $x=\left(\dfrac{25+16w}{80}\right)^2$ Or equivalent.	• Reaching stage #(1).	• Correct expression for x.	

(d)	$\frac{1}{x+y\mathrm{i}}=1-\frac{1}{1+p\mathrm{i}}$ $\frac{1}{x+y\mathrm{i}}=\frac{1+p\mathrm{i}-1}{1+p\mathrm{i}}$ $\frac{1}{x+y\mathrm{i}}=\frac{p\mathrm{i}}{1+p\mathrm{i}}$ #(1) $x+y\mathrm{i}=\frac{1+p\mathrm{i}}{p\mathrm{i}}$ $x+y\mathrm{i}=\frac{(1+p\mathrm{i})\mathrm{i}}{p\mathrm{i}\times\mathrm{i}}$ $x+y\mathrm{i}=\frac{\mathrm{i}-p}{-p}$ $x+y\mathrm{i}=1-\frac{1}{p}\mathrm{i}$ So $x=1$ and $y=-\frac{1}{p}$	• Reaching stage #(1).	• Correct values for x and y.	
(e)	$z+2\mathrm{i}=\mathrm{i}z+k$ $z-\mathrm{i}z=k-2\mathrm{i}$ $z(1-\mathrm{i})=k-2\mathrm{i}$ $z=\frac{k-2\mathrm{i}}{1-\mathrm{i}}$ #(1) Then $w=z(2+2\mathrm{i})$ $w=\frac{k-2\mathrm{i}}{1-\mathrm{i}}\times(2+2\mathrm{i})$ $w=\frac{(k-2\mathrm{i})(2+2\mathrm{i})(1+\mathrm{i})}{(1-\mathrm{i})(1+\mathrm{i})}$ $w=\frac{(k-2\mathrm{i})(2+4\mathrm{i}+2\mathrm{i}^2)}{1-\mathrm{i}^2}$ $w=\frac{(k-2\mathrm{i})4\mathrm{i}}{2}$ #(2) $w=(k-2\mathrm{i})2\mathrm{i}$ $w=2k\mathrm{i}-4\mathrm{i}^2$ $w=4+2k\mathrm{i}$ #(3) Therefore $\mathrm{Im}(w)=8\Rightarrow 2k=8$ $k=4$	• Reaching stage #(1).	• Reaching stage #(2).	E 7 Reaching stage #(3) . OR Correct solution but with one minor error. E 8 Correct value of k, with valid and clear justification.

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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 12	13 – 18	19 – 24