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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

SUPERVISOR'S USE ONLY

Level 3 Calculus, 2013

91577 Apply the algebra of complex numbers in solving problems

9.30 am Wednesday 13 November 2013
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–10 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

You are advised to spend 60 minutes answering the questions in this booklet.

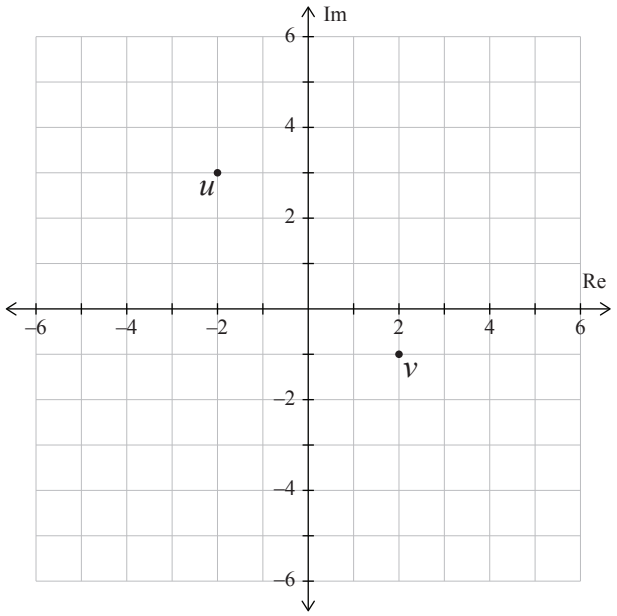
ASSESSOR'S USE ONLY

QUESTION ONE

(a) Write $\dfrac{4}{2-\sqrt{5}}$ in the form $a+b\sqrt{5}$, where a and b are rational numbers.

(b) Complex numbers u and v are represented on the Argand diagram.

If $w = u - v$, then show w on the Argand diagram below.



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(c) One root of the equation $z^3 - 3z + p = 0$ is $z = 2 - 3\text{i}$.

If p is a real number, find the value of p and the other roots of the equation.

(d) If $z = 1 + \text{i}$ and $w = \frac{1}{z} + \text{i}$, find the exact value of $\arg(w)$.

(e) If $u = 6 + \text{ki}$ and $v = 4 + \text{ki}$, find k if $\arg(u.v) = \frac{\pi}{4}$.

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QUESTION TWO

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(a) What is the remainder when $x^3 + 4x^2 + 3x - 9$ is divided by $x + 2$?

(b) If $u = 2\text{cis}\frac{2\pi}{3}$ and $v = 6\text{cis}\frac{\pi}{2}$, write $\frac{u}{v}$ in polar form.

(c) Find the equation whose roots are three times those of $x^2 + 9x - 12 = 0$.

(d) If $u = x + iy$ and $au^2 + bu + c = 0$, prove that $a\bar{u}^2 + b\bar{u} + c = 0$.

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(e) Describe fully the locus of the points representing z if $\dfrac{z+2\mathrm{i}}{z-2\mathrm{i}}$ is purely imaginary.

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QUESTION THREE

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(a) Solve the equation $z^2 + 6z + 20 = 0$.

Express the solutions in the form $z = a + \sqrt{b}\,\mathrm{i}$, where a and b are integers.

(b) p and q are complex numbers, where $p = 3 + 4\mathrm{i}$ and $q = 2 - 3\mathrm{i}$.

Find $p\bar{q}$, expressing your answer in the rectangular form $a + b\,\mathrm{i}$.

(c) Solve the following equation for x in terms of p:

$$\sqrt{x} - 3 = \sqrt{x - p}$$

(d) Find all the solutions of the equation $z^3 + n = 0$, where n is a positive real number.
Write your solutions in polar form as expressions in terms of n.

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(e) Find the values of k for which the equation $6 + x - 4\sqrt{3x + k} = 0$ has no real roots.

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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