3

91577

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Level 3 Calculus, 2014

91577 Apply the algebra of complex numbers in solving problems

9.30 am Tuesday 18 November 2014
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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(a) Given that $x - 2$ is a factor of $g(x) = x^3 - 2px^2 + px - 5$, find the value of p where p is real.

(b) If $u = 3 - 3\text{i}$, find u^4 in the form $r\operatorname{cis}\theta$.

(c) Solve the equation $x = \sqrt{33 - 4x} + 3$.

(d) Solve the equation $z^4 = -4k^2\text{i}$, where k is real.

Write your solutions in polar form in terms of k.

(e) Find the equation of the locus described by $|z - 1 + 2\text{i}| = |z + 1|$.

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QUESTION TWO

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(a) Solve the equation $x^2 - 4x + 16 = 0$.

Give your solutions in the form $a \pm \sqrt{b}\,\mathrm{i}$, where a and b are rational numbers.

(b) Given that $w = 2\operatorname{cis}\frac{\pi}{3}$, find w^4.

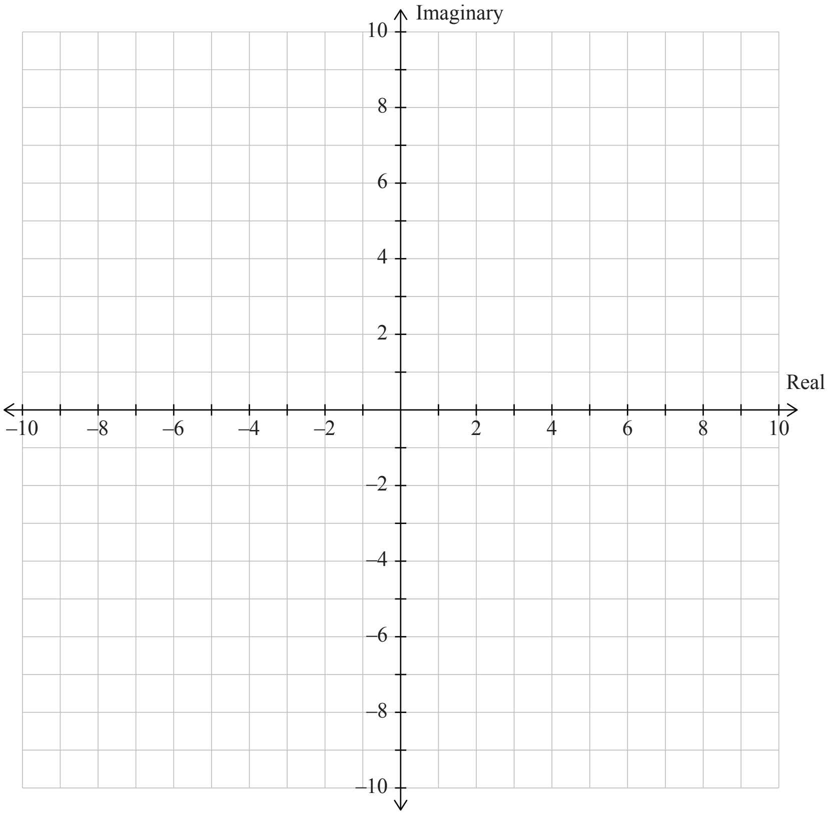
Give your answer in the form $a + b\mathrm{i}$, where a and b are real.

(c) $w = 2 - 3\mathrm{i}$ is a solution of the equation $3w^3 - 14w^2 + Aw - 26 = 0$, where A is real.

Find the value of A and the other two solutions of the equation.

(d) A complex number z satisfies $|z - 3 - 4i| = 2$.

(i) Sketch the locus of points that represents z on the Argand diagram below



(ii) What is the maximum value of Re(z)?

(e) The complex number z is given by $z = \frac{1+3\mathrm{i}}{p+q\mathrm{i}}$, where p and q are real and $p > q > 0$.

Given that $\mathrm{Arg}(z) = \frac{\pi}{4}$, show that $p - 2q = 0$.

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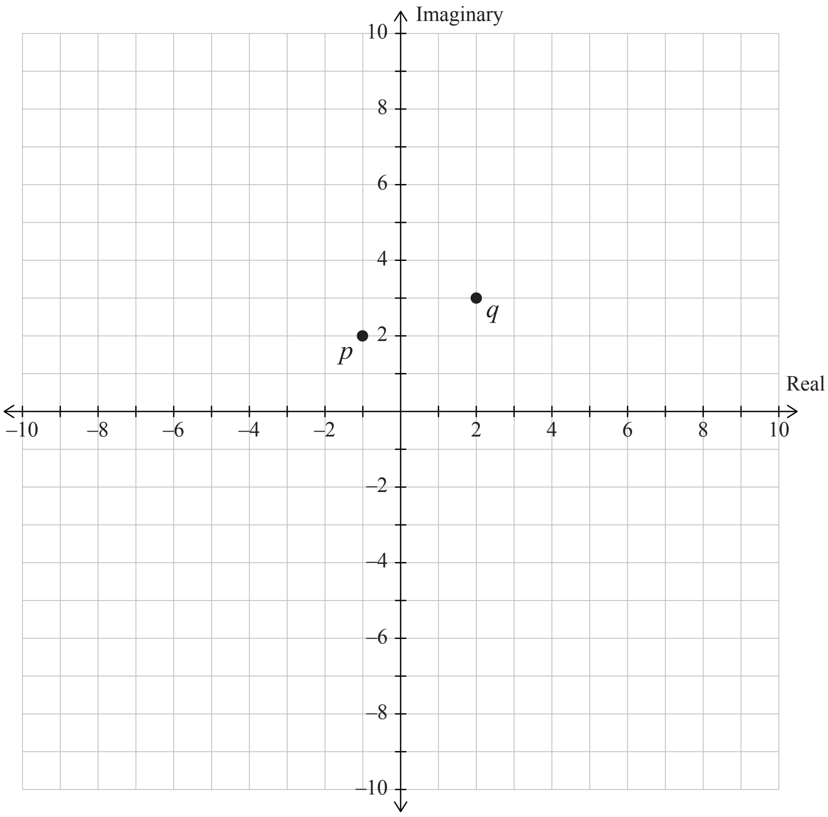
QUESTION THREE

(a) Expand and simplify as far as possible the following expression:

$$\left(2-\sqrt{3}\right)\left(5+2\sqrt{3}\right)\left(4-3\sqrt{3}\right)$$

Give your answer in the form $a+b\sqrt{3}$, where a and b are real numbers.

(b) The complex numbers p and q are represented on the Argand diagram below.



If $r = 2p - 3q$, find r and mark it on the Argand diagram above.

(c) For what values of p, where p is real, does the graph of $y = px^2 - 4px + 1$ **not** intersect the x-axis?

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(d) Given that $z = 3 + 2\mathrm{i}$, find the value of $\overline{z}^2 + \frac{1}{z^2}$, giving your answer in the form $a + b\mathrm{i}$, where a and b are real.

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(e) α, β, and γ are the three roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$, where a, b, c, and d are real numbers.

(i) Prove that

$$\alpha+\beta+\gamma=\frac{-b}{a}, \quad \alpha\beta+\beta\gamma+\alpha\gamma=\frac{c}{a}, \quad \alpha\beta\gamma=\frac{-d}{a}$$

(ii) Hence prove that $\alpha^2\beta\gamma+\alpha\beta^2\gamma+\alpha\beta\gamma^2=\dfrac{bd}{a^2}$

Extra paper if required.
Write the question number(s) if applicable.

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