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SUPERVISOR'S USE ONLY

Level 3 Calculus, 2015

91577 Apply the algebra of complex numbers in solving problems

2.00 p.m. Wednesday 25 November 2015
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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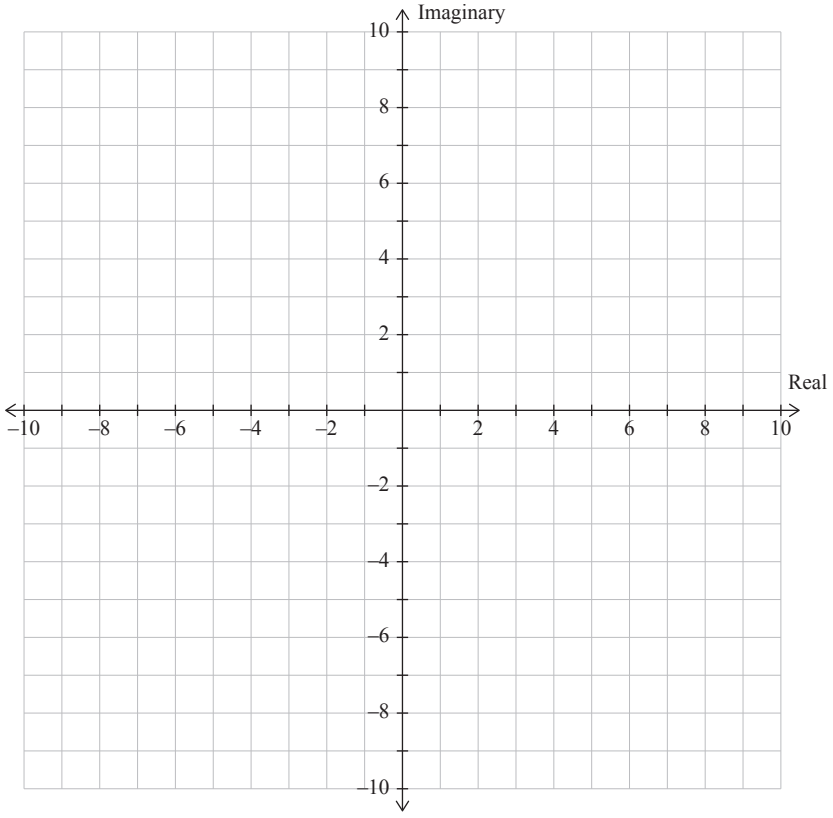
(a) Solve the equation $x^2 - 8x + 4 = 0$.

Write your answer in the form $a \pm b\sqrt{c}$, where a, b, and c are integers and $b \neq 1$.

(b) If $u = 1 + \sqrt{3}\mathrm{i}$, clearly show u^3 on the Argand diagram below.



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(c) v is the complex number $3 - 7i$
w is the complex number $-4 + 6i$.

Find the real numbers p and q such that $pv + qw = 6.5 - 11i$.

(d) Prove that the roots of the equation $3x^2 + (2c + 1)x - (c + 3) = 0$ are always real for all values of c, where c is real.

(e) If $x^2 + bx + c$ and $x^2 + dx + e$ have a common factor of $(x - p)$,

prove that $\dfrac{e-c}{b-d} = p$, where $b, c, d, e,$ and p are all real.

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QUESTION TWO

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(a) What is the remainder when $2x^3 + x^2 - 5x + 7$ is divided by $x + 3$?

(b) The complex number $\dfrac{2+3\mathrm{i}}{5+\mathrm{i}}$ can be expressed in the form $k(1 + \mathrm{i})$, where k is a real number.

Find the value of k.

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(c) Find real numbers A, B and C such that $\dfrac{1}{x^2(x-1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{(x-1)}$

(d) Write the complex number $\left(\dfrac{4\mathrm{i}^7 - \mathrm{i}}{1+2\mathrm{i}}\right)^2$ in the form $a + b\mathrm{i}$, where a and b are real numbers.

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(e) Find the Cartesian equation of the locus described by $\arg\left(\frac{z-2}{z+5}\right)=\frac{\pi}{4}$

QUESTION THREE

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(a) If $z = 4 + 2i$ and $w = -1 + 3i$, find $\arg(zw)$.

(b) For what real value(s) of k does the equation $kx^2 + \frac{x}{k} + 2 = 0$ have equal roots?

(c) One solution of the equation $3w^3 + Aw^2 - 3w + 10 = 0$ is $w = -2$.

If A is a real number, find the value of A and the other two solutions of the equation.

(d) Solve the equation $z^3 = k + \sqrt{3}\,k\mathrm{i}$, where k is real and positive.
Write your solutions in polar form in terms of k.

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Question Three continues on the following page.

(e) (i) Find each of the roots of the equation $z^5 - 1 = 0$.

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(ii) Let p be the root in part (i) with the smallest positive argument.

Show that the roots in part (i) can be written as $1, p, p^2, p^3, p^4$.

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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