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QUALIFY FOR THE FUTURE WORLD
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Level 3 Calculus, 2018

91577 Apply the algebra of complex numbers in solving problems

9.30 a.m. Tuesday 13 November 2018
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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(a) What is the remainder when $2x^3 - 3x^2 + 4x + 3$ is divided by $x - 2$?

(b) If $u = m\,\text{cis}\,\frac{\pi}{3}$ and $v = m^3\text{cis}\,\frac{2\pi}{5}$, find uv in polar form.

(c) Solve the equation $2 + \sqrt{x} = \sqrt{x + k}$ for x in terms of k.

(d) Find the exact value(s) of k for which the equation $k(1 + x^2) = 3 - 8x - x^2$ has one repeated solution.

Give your solution in the form $k = a \pm \sqrt{b}$.

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(e) If $z = a + b\text{i}$ and $\frac{z}{\bar{z}} = c + d\text{i}$, prove that $c^2 + d^2 = 1$.

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QUESTION TWO

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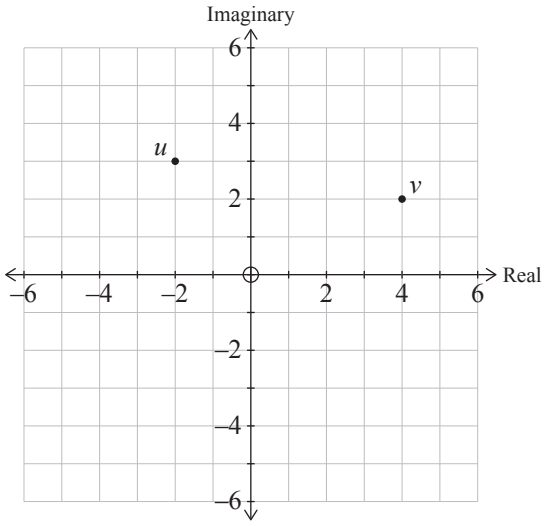
(a) Complex numbers u and v are represented on the Argand diagram below.

If $w = u + \bar{v}$, show w on the Argand diagram.



(b) Write $\dfrac{6}{3-\sqrt{7}}$ in the form $a + b\sqrt{7}$.

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(c) One solution of the equation $z^3 + Az^2 + 34z - 40 = 0$ is $z = 3 + i$.

If A is a real number, find the value of A and the other two solutions of the equation.

(d) If $z = \frac{15}{1-2i} - 2i$, find mod(z).

You must show all algebraic working.

(e) The complex number $u = 3 + mi$ is on the locus of points defined by $|z - 8| = |z - 4 + 2i|$.

Find the value of m.

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QUESTION THREE

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(a) $u = 3 - 2i$ and $v = 2 + bi$.

Find the value of b if $uv = 14 + 8i$

(b) Solve the equation $x^2 - 6px + 4p^2 = 0$ for x in terms of p, expressing the solution in its simplest form.

(c) Solve the equation $z^3 = -k^6 i$, where k is real and positive.

Write your solutions in polar form in terms of k.

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(d) Find the complex number w, in the form $x + iy$, if $\arg(w) = \frac{\pi}{4}$ and $|w \cdot \overline{w}| = 20$.

(e) Solve the equation $\frac{\sqrt{x+k} + \sqrt{x-k}}{\sqrt{x+k} - \sqrt{x-k}} = 4$ for x in terms of k.

Extra paper if required.
Write the question number(s) if applicable.

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