91577

915770

3

SUPERVISOR'S USE ONLY

NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 3 Calculus, 2019

91577 Apply the algebra of complex numbers in solving problems

9.30 a.m. Tuesday 26 November 2019
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

(a) Solve the equation $x^2 - 4x + 7 = 0$.

Give your solution in the form $a \pm \sqrt{b}\,\mathrm{i}$, where a and b are rational numbers.

(b) When the polynomial $2x^3 - x^2 - 4x + p$ is divided by $x - 3$, the remainder is 38.

Find the value of p.

(c) Complex numbers u and v are $u = q + 2\mathrm{i}$ and $v = 1 - 2\mathrm{i}$.

Given that $\left|\frac{u}{v}\right| = 13$, find all possible values of q.

(d) One solution of the equation $2z^3 - 5z^2 + cz - 5 = 0$ is $z = 1 - 2i$.

If c is real, find the value of c and the other two solutions of the equation.

ASSESSOR'S USE ONLY

ASSESSOR'S USE ONLY

(e) Find the values of x and y, given that x and y are real, and $\dfrac{1}{x+\mathrm{i}y}-\dfrac{1}{1+\mathrm{i}}=1-2\mathrm{i}$.

QUESTION TWO

ASSESSOR'S USE ONLY

(a) If $p = 3 - i$ and $q = -2 + 5i$, find $\overline{p} - 3q$, giving your solution in the form $a + bi$.

(b) Write $\dfrac{3}{4-\sqrt{5}}$ in the form $a + b\sqrt{5}$ where a and b are rational numbers.

(c) Solve the equation $z^4 + 16p^2i = 0$, where p is real.
Write your solution in polar form, in terms of p.

ASSESSOR'S USE ONLY

(d) Find all possible values of m that make $z = \dfrac{\sqrt{3} + m\mathrm{i}}{1 + \sqrt{3}\,\mathrm{i}}$ a purely real number.

(e) If $|z| = 1$, and $z \neq 1$, prove that $\dfrac{1+z}{1-z}$ is purely imaginary.

ASSESSOR'S USE ONLY

QUESTION THREE

ASSESSOR'S USE ONLY

(a) If $u = q^2\text{cis}\frac{3\pi}{4}$ and $v = q^3\text{cis}\frac{\pi}{3}$, write $\frac{u}{v}$ in the form $r\,\text{cis}\,\theta$.

(b) If x and y are real numbers and $(x + iy)(2 + i) = 3 - i$, find the values of x and y.

(c) Solve the following equation for x in terms of w.

$$2\sqrt{x-3} - w\sqrt{x} = 0$$

(d) Two complex numbers are defined by $u = 1 + pi$ and $v = 5 + 3i$.

Given that $\arg\left(\frac{u}{v}\right) = \frac{\pi}{4}$, find the value of p.

ASSESSOR'S USE ONLY

Question Three continues on the following page.

(e) Prove that the quadratic equation $x^2 + 3kx + k^2 = 7x + 3k$ will have two distinct real solutions for all real values of k.

ASSESSOR'S USE ONLY

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

91577