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Level 3 Calculus 2020

91577 Apply the algebra of complex numbers in solving problems

9.30 a.m. Monday 23 November 2020
Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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(a) If $s = 2 + 3\text{i}$ and $t = 3 + k\text{i}$, find the value of k if $st = 21 - \text{i}$.

(b) Find the value(s) of r such that the equation $x^2 + 4rx + r = 0$ has only one solution.

(c) Solve the following equation for x in terms of g.

$$2\sqrt{x} - 5 = \sqrt{4x - g}$$

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(d) Write $\dfrac{k+k\mathrm{i}}{1-\mathrm{i}}+\dfrac{2k}{1+\mathrm{i}}$ in its simplest possible form.

(e) Given that $T=\dfrac{a-b\mathrm{i}}{a+b\mathrm{i}}$, where a and b are real constants, prove that $\dfrac{1+T^2}{2T}=\dfrac{a^2-b^2}{a^2+b^2}$.

QUESTION TWO

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(a) Given that $x - 2$ is a factor of $2x^3 + qx^2 - 17x - 10$, find the value of q.

(b) Find all possible values of k given that $|5 + 3k\text{i}| = 13$.

(c) One of the solutions of $2z^3 - 15z^2 + bz - 30 = 0$ is $z = 3 + \text{i}$ (b is a real number).

Find the other solutions, and the value of b.

(d) Given that $u = p + p\text{i}$ and $v = -q + q\text{i}$, where p and q are both positive real constants, find $\arg\left(\dfrac{u}{v}\right)$.

(e) Find the Cartesian equation of the locus described by $|z + \text{i}|^2 + |z - \text{i}|^2 = 10$.
Write your solution in the form $x^2 + y^2 = k$.

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QUESTION THREE

(a) If $u = 12k^3\text{cis}(\pi)$ and $v = 2k\text{cis}\left(\frac{\pi}{3}\right)$, write the exact value of $\frac{u}{v}$ in polar form.

(b) If $z = 5 - \text{i}$ and $w = -2 + 3\text{i}$, show that $|z|^2 = 2|w|^2$.

(c) Given that $z = a + b\text{i}$, where a and b are non-zero real numbers,
show that $\frac{z\overline{z}}{z+\overline{z}}$ is a real number.

(d) Solve the equation $z^4 = -16k^8$, where k is a real constant.

Give your solutions in polar form in terms of k.

(e) For complex numbers u and v, prove that if $|u + v| = |u - v|$, then $\frac{u}{v}$ is purely imaginary.

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Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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