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Level 3 Calculus 2021

91577 Apply the algebra of complex numbers in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2 – 12 in the correct order and that none of these pages is blank.

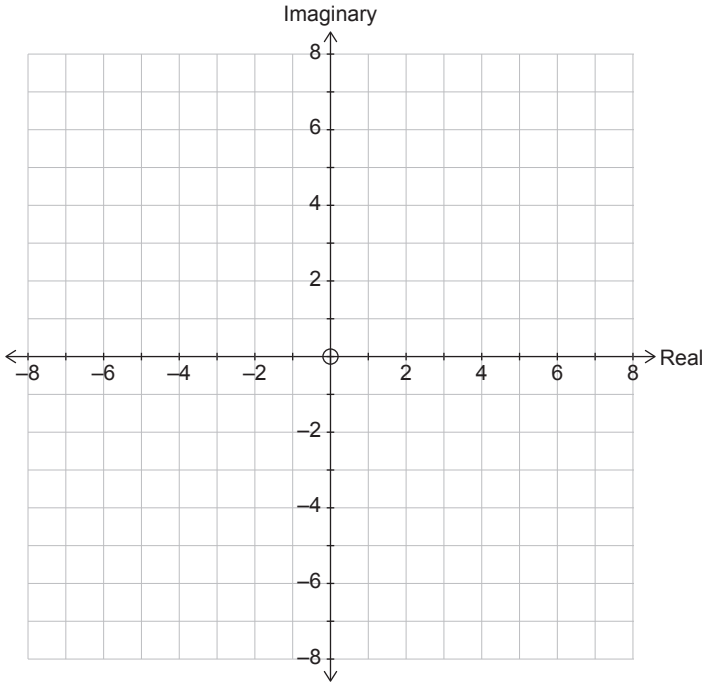
Do not write in any cross-hatched area (▨). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) Given that $w = d + 5i$ and $z = 3 - 4i$, find the value of d if $wz = 38 - 9i$.

(b) If $z = 2 + 3i$, show $\frac{26}{z}$ on the Argand diagram below.



(c) The polynomial $f(x) = x^3 + 3x^2 + ax + b$ has the same remainder when divided by $(x - 2)$ as it does when divided by $(x + 1)$.

The polynomial $f(x)$ also has $(x + 2)$ as a factor.

Find the values of a and b.

(d) Show that if $z = 1 + 3\text{i}$, then $\arg\left(\dfrac{z-1}{z-2\text{i}}\right) = \dfrac{\pi}{4}$.

(e) Given that the real part of $\frac{z-2i}{z-4}$ is zero and $z \neq 4$, prove that the locus of points described by z is given by the Cartesian equation $(x-2)^2 + (y-1)^2 = 5$.

QUESTION TWO

(a) Given that $u = 2\text{i}$ and $w = 2\text{cis}\left(\frac{2\pi}{3}\right)$, find $z = \frac{u}{w}$.

(b) Solve the equation $x^2 - 12qx + 20q^2 = 0$ for x in terms of q, expressing any solutions in their simplest form.

(c) Prove that $\frac{a+b\text{i}}{b-a\text{i}}$ is purely imaginary, where a and b are real constants.

(d) Solve the equation $z^3 = k^6 + k^6 i$, where k is a real constant.

(e) If z is a complex number and $|z + 16| = 4|z + 1|$, find the value of $|z|$.

QUESTION THREE

(a) The complex number $u = 5 + m\text{i}$ has $|u| = 6$.

Given that $0 < \arg(u) < \frac{\pi}{2}$, find the exact value of real number m.

(b) Write $\frac{18}{4 - 2\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers.

Question Three continues on the next page.

(c) One solution of $4z^3 - 19z^2 + 128z + A = 0$ is $z = 2 + 5i$.

If A is real, find the value of A and the other two solutions of the equation.

(d) Solve the following equation for x in terms of m.

$$6\sqrt{2x} - 5 = 6\sqrt{2x + m}$$

(e) Solve the equation $z^2 = i(|z|^2 - 4)$.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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QUESTION NUMBER