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Level 3 Calculus 2022

91577 Apply the algebra of complex numbers in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (⬚). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) Write $\dfrac{12k}{1+\sqrt{5}}$ in the form $ak + bk\sqrt{5}$, where a and b are integers.

(b) If $u = m^5\mathrm{cis}\dfrac{\pi}{3}$ and $v = m^2\mathrm{cis}\dfrac{\pi}{5}$, write $\dfrac{u}{v}$ in polar form.

(c) If $u = 3 + 2i$, $v = 4 + 2i$, and $w = 2 + ki$, find the value of k if $\arg(uvw) = \dfrac{\pi}{4}$.

(d) Find the value(s) of p for which the equation $x - 2\sqrt{x+p} = -5$ has only one real solution.

(e) For complex numbers w and z, prove that:

$$\left|w+z\right|^2 - \left|w-\overline{z}\right|^2 = 4\,\mathrm{Re}(w)\,\mathrm{Re}(z)$$

where $\mathrm{Re}(w)$ is the real part of w, and $\mathrm{Re}(z)$ is the real part of z.

QUESTION TWO

(a) Dividing $x^3 - 3x^2 + bx + 9$ by $(x+2)$ gives a remainder of 3.

Find the value of b.

(b) Find the complex number z for which $z + 4\bar{z} = 15 + 12i$.

(c) One of the solutions of $z^3 - 2z^2 + hz + 180 = 0$ is $z = -4$ (h is a real number).

Find the other solutions, in the form $a \pm bi$, and the value of h.

(d)   If $z = 1 - \sqrt{3}i$ and $w = \dfrac{4}{z} - 2$, find $\arg(w)$.

(e)   Find the Cartesian equation of the locus described by $|z + i| = 2|z - 5i|$
      in the form $(x - a)^2 + (y - b)^2 = k^2$.

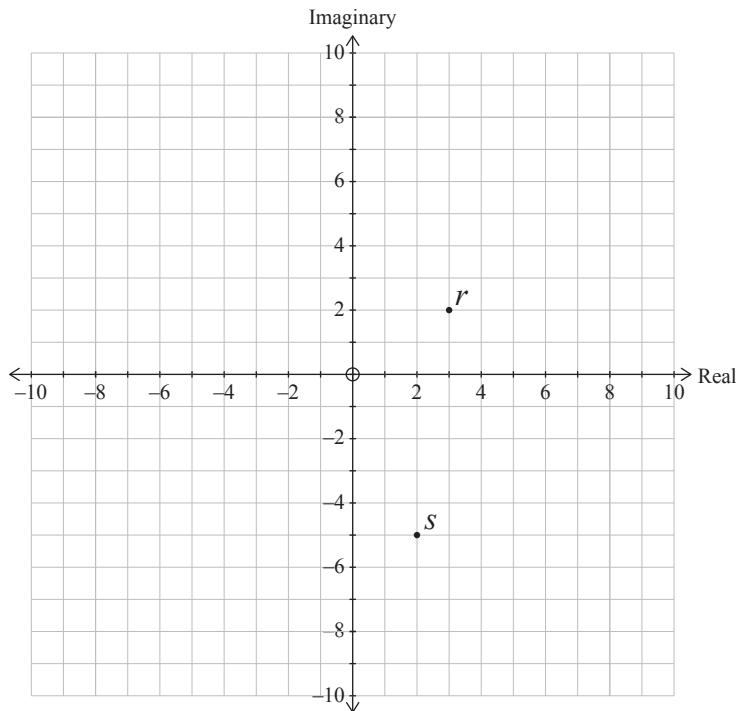**QUESTION THREE**

(a) The complex numbers r and s are represented on the Argand diagram below.

If $v = 2r - s$, find v and mark it on the Argand diagram above.

(b) Solve the equation $z^2 + 6kz + 15k^2 = 0$ in terms of real number k.

Give your solution in the form $ak \pm \sqrt{bk}i$, where a and b are rational numbers.

(c) Solve the equation $z^3 + k^6 i = 0$, where k is a real constant.
Give your solution(s) in polar form in terms of k.

(d) Prove that there is no complex number z such that $|z| - z = i$.

(e) If $z = a + bi$ is a non-zero complex number, and $\dfrac{i}{z} + \dfrac{3}{\overline{z}} = 1$, find the values of a and b.

QUESTION NUMBER	

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