

SUPERVISOR'S USE ONLY

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Mana Tohu Mātauranga o Aotearoa  
New Zealand Qualifications Authority

## Level 3 Calculus 2023

### 91577 Apply the algebra of complex numbers in solving problems

Credits: Five

Achievement	Achievement with Merit	Achievement with Excellence
Apply the algebra of complex numbers in solving problems.	Apply the algebra of complex numbers, using relational thinking, in solving problems.	Apply the algebra of complex numbers, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

**You should attempt ALL the questions in this booklet.**

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (DO NOT WRITE). This area will be cut off when the booklet is marked.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

**QUESTION ONE**

(a) Write  $(5 - 2\sqrt{p})^2$  in the form  $a + bp + c\sqrt{p}$  where  $a$ ,  $b$ , and  $c$  are integers.

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(b) Find the value(s) of  $r$  so that the quadratic equation  $4x^2 - 4x + 3r - 2 = 0$  has no real roots.

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(c) If  $z = p + qi$  and  $w = a + bi$  and  $\operatorname{Re}\left(\frac{z}{w}\right) = 0$ , then show that  $ap = -bq$ .

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## QUESTION TWO

- (a) If  $u = q^6 \operatorname{cis} \frac{5\pi}{8}$  and  $v = q^2 \operatorname{cis} \frac{2\pi}{5}$ , write  $\frac{u}{v}$  in the form  $r \operatorname{cis} \theta$ .

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- (b) If  $z = 1 + ki$  and  $w = 7 - ki$ , then find  $|z - w|$ , giving your answer in terms of  $k$ .

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- (c) Find  $\operatorname{Arg}(z)$  if  $\frac{13z}{z+1} = 11 - 3i$ .

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### QUESTION THREE

- (a) When the polynomial  $2x^3 + px^2 + 7x - 3$  is divided by  $x + 3$ , the remainder is 30.

Find the value of  $p$ .

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- (b) The complex numbers  $u$  and  $v$  are  $u = n - i$  and  $v = 2 - 3i$ .

Given that  $\frac{u}{v} = 3 + 4i$ , find the value of  $n$ .

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- (c) Solve the following equation for  $x$ , in terms of  $w$ .

$$4\sqrt{4x - w} = 5 - 8\sqrt{x}$$

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Extra space if required.  
Write the question number(s) if applicable.

QUESTION  
NUMBER

