Assessment Schedule – 2013

Calculus: Apply differentiation methods in solving problems (91578)

Evidence Statement

One	Expected Coverage	Achievement	Merit	Excellence
(a)	$\frac{dy}{dx} = \sec^2(x^2+1)\cdot 2x$	Correct derivative.		
(b)	$\frac{dy}{dx} = \frac{3-e^x}{3x-e^x}$ or no tangent exists At $x = 0$ gradient = –2	Correct solution with correct derivative shown.		
(c)	$\frac{dy}{dx} = -2xe^{6-x^2}$ $\frac{d^2y}{dx^2} = -2e^{6-x^2} + 4x^2e^{6-x^2}$ Point of inflection when $\frac{d^2y}{dx^2} = 0$ $(4x^2-2)e^{6-x^2} = 0$ $4x^2 - 2 = 0$ $x = \pm\frac{1}{\sqrt{2}}$	Correct $\frac{dy}{dx}$	Correct solution with correct first and second derivatives. $\pm$ not required, accept positive answer only.	
(d)	$\frac{dx}{dt} = 5\cos t$ $\quad \frac{dy}{dt} = 3\sec^2 t = \frac{3}{\cos^2 t}$ $\frac{dy}{dx} = \frac{3}{5\cos^3 t}$ At $t = \frac{\pi}{3}$, $\frac{dy}{dx} = \frac{24}{5}(= 4.8)$ $\therefore$ gradient of normal = $\frac{-5}{24}(= -0.2083)$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$.	Correct solution including all correct derivatives.	

(e)	$20 = 2\pi r^2 + 2\pi rh$ $2\pi r(r+h) = 20$ $h = \frac{10}{\pi r} - r$ $V = \pi r^2 h = \pi r^2 \cdot \left(\frac{10}{\pi r} - r\right)$ $V = 10r - \pi r^3$ $\frac{dV}{dr} = 10 - 3\pi r^2$ $\frac{dV}{dr} = 0 \Rightarrow r = \sqrt{\frac{10}{3\pi}}$ or $r = 1.03$ m OR $20 = \pi r^2 + 2\pi rh$ $V = 10r - \frac{\pi r^3}{2}$ $\frac{dV}{dr} = 10 - \frac{3\pi r^2}{2}$ $r = \sqrt{\frac{20}{3\pi}} = 1.46$		Equation for volume in terms of 1 variable found, and differentiated correctly.	Problem solved including correct derivative.

NØ = No response / no relevant evidence
N1 = ONE question demonstrating limited knowledge of differentiation techniques
N2 = ONE correct derivative
A3 = TWO of Achievement
A4 = THREE of Achievement
M5 = ONE of Merit
M6 = TWO of Merit
E7 = Excellence with minor errors ignored
E8 = Excellence correct

Two	Expected Coverage	Achievement	Merit	Excellence
(a)	$\frac{dy}{dx}=\frac{1}{3}\left(\pi-x^2\right)^{\frac{-2}{3}}\cdot -2x$ or $\frac{dy}{dx}=\frac{-2x}{3\left(\pi-x^2\right)^{\frac{2}{3}}}$	Correct derivative.		
(b)	$\frac{dy}{dx}=3\left(x^3-2x\right)^2\cdot\left(3x^2-2\right)$ At $x = 1$, $\frac{dy}{dx}=3\cdot(-1)^2\cdot 1=3$ At $x = 1$, $y=-1$ $y+1=3(x-1)$ $y=3x-4$	Correct solution with correct derivative shown.		
(c)	$f'(x)=1-e^x+\frac{k}{x^2}$ $f'(x)=0 \quad \Rightarrow 1-e^{-1}+k=0$ $k=e^{-1}-1$ Or $k=-0.632$	Correct derivative.	Correct value for k and correct derivative.	
(d)(i) (ii) (iii)	1. $x = 1$ 2. $x>3$ 3. $-2, -1, 3$ -3 Does not exist.		THREE correct answers (out of 5).	
(e)	$A(\theta)=64\sin\theta+64\sin\theta\cos\theta$ OR $A(\theta)=64\sin\theta+32\sin 2\theta$ $A'(\theta)=64\cos\theta+64\cos^2\theta-64\sin^2\theta$ OR $A'(\theta)=64\cos\theta+64\cos 2\theta$ $=64\cos\theta+64\cos^2\theta-64(1-\cos^2\theta)$ $=64(2\cos^2\theta+\cos\theta-1)$ Minimum when $A'(\theta)=0$ $2\cos^2\theta+\cos\theta-1=0$ $\left(2\cos\theta-1\right)\left(\cos\theta+1\right)=0$ Or $\cos\theta=\frac{1}{2}$ or $\cos\theta=-1$ (NO) $\theta=60°$ or $\theta=\frac{\pi}{3}$		Correct derivative.	Correct solution with correct derivatives.

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Three	Expected Coverage	Achievement	Merit	Excellence
(a)	$\frac{dy}{dx}=\frac{x^2\cdot\cos 2x\cdot 2-2x\sin 2x}{x^4}$	Correct derivative.		
(b)	$f'(x)=1-16(x-2)^{-2}$ Turning point when $f'(x)=0$ $1=\frac{16}{(x-2)^2}$ $(x-2)^2=16$ $x=-2$ or $x=6$	Correct solution with correct derivative.		
(c)	$f'(x)=50-\left(30\ln 2x+30x\cdot\frac{1}{x}\right)$ $=20-30\ln 2x$ Maximum when $f'(x)=0$ $20=30\ln 2x$ $\frac{2}{3}=\ln 2x$ $x=\frac{e^{\frac{2}{3}}}{2}=0.974$	Correct derivative.	Correct solution with correct derivative.	
(d)	For the curve, $\frac{dy}{dx}=\frac{3t^2-3}{2t-1}$ Normal parallel to the y-axis means tangent parallel to the x-axis. $\Rightarrow\frac{dy}{dx}=0$ $3t^2=3$ $t=\pm 1$ $t=1\Rightarrow$ point (0,–2) $t=-1\Rightarrow$ point (2,2)	Correct expression for $\frac{dy}{dx}$	Correct solution with correct derivative.	

(e)	$\frac{dV}{dt} = 300$ $A = 4\pi r^2 \quad \Rightarrow \frac{dA}{dr} = 8\pi r$ $V = \frac{4}{3}\pi r^3 \quad \Rightarrow \frac{dV}{dr} = 4\pi r^2$ $\frac{dA}{dt} = \frac{dV}{dt} \cdot \frac{dA}{dr} \cdot \frac{dr}{dV}$ $= \frac{2400\pi r}{4\pi r^2}$ $= \frac{600}{r}$ A = 7500 $\Rightarrow 4\pi r^2 = 7500$ $r = \sqrt{\frac{7500}{4\pi}} = 24.43$ cm $\therefore \frac{dA}{dt} = \frac{600}{24.43} = 24.56 \text{ cm}^2 \text{ s}^{-1}$	Correct expressions for $\frac{dV}{dr}$ and $\frac{dA}{dr}$	Correct expressions for $\frac{dV}{dr}$, $\frac{dA}{dr}$ and $\frac{dA}{dt}$	Correct solution along with correct expressions for $\frac{dV}{dr}$, $\frac{dA}{dr}$ and $\frac{dA}{dt}$

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Judgement Statement

	Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
Score range	0 – 8	9 – 13	14 – 20	21 – 24