Assessment Schedule – 2015

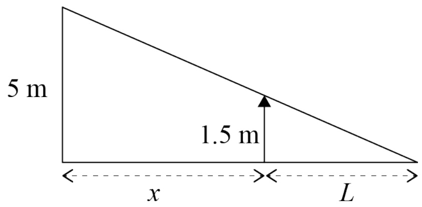
Calculus: Apply differentiation methods in solving problems (91578)

Evidence

Q1	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	$30\sec^2(5x)$	A correct expression for the derivative.		
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3(4x-3x^2)^2(4-6x)$ At $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 3\times 1\times -2 = -6$	Correct solution with correct derivative.		
(c)	$f'(x) = 8 - \frac{2}{(x+1)^2}$ $f'(x) > 0 \Rightarrow 8 > \frac{2}{(x+1)^2}$ $(x+1)^2 > \frac{1}{4}$ Either $x+1 > \frac{1}{2}$ or $x+1 < \frac{-1}{2}$ $x > \frac{-1}{2}$ or $x < \frac{-3}{2}$	Correct derivative.	Correct solution with correct derivative.	
(d)	$f'(x) = \frac{x(x-5)-(x+4)(2x-5)}{x^2(x-5)^2}$ $f'(x) = 0 \Rightarrow x(x-5)-(x+4)(2x-5) = 0$ $x^2-5x-(2x^2+3x-20) = 0$ $-x^2-8x+20 = 0$ $x^2+8x-20 = 0$ $(x+10)(x-2) = 0$ $x = -10$ or $+2$	Correct derivative.	Correct solution with correct derivative.	

(e)	Let V = volume (m^3) S = slant height (m) h = height (m) r = radius (m) $\cos 30 = \frac{r}{S}$ $S = \frac{r}{\cos 30}$ $\frac{dS}{dr} = \frac{1}{\cos 30}$ $\tan 30 = \frac{h}{r}$ $h = r\tan 30$ $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi r^3 \tan 30$ $\frac{dV}{dr} = \pi r^2 \tan 30$ $\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$ $= \frac{1}{\cos 30} \times \frac{1}{\pi r^2 \tan 30} \times 2$ When $r = 10$ m, $\frac{dS}{dt} = \frac{1}{\cos 30} \times \frac{1}{\pi 10^2 \times \tan 30} \times 2$ $= 0.01273$ m / minute	$\frac{dS}{dr}$ or $\frac{dV}{dr}$ correct.	Valid statement of the relationship between rates.	Correct solution with correct derivatives.

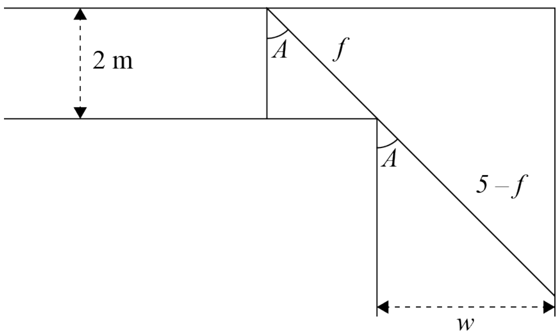
NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q2	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	$\frac{1}{5}\left(x-3x^2\right)^{\frac{-4}{5}}\cdot\left(1-6x\right)$	A correct expression for the derivative.		
(b)	$\frac{dy}{dx}=1+\frac{16}{x^2}$ At $x=4$, $\frac{dy}{dx}=2$ $\therefore$ Gradient of normal $=\frac{-1}{2}$	Correct solution with correct derivative.		
(c)(i) (ii) (iii)	1. $x=1$ 2. $x=-1, 1, 2$ 3. $-1<x<1$ 3 Does not exist.	Two correct answers.	Four correct answers.	
(d)	 $\frac{x+L}{5}=\frac{L}{1.5}$ $1.5x+1.5L=5L$ $1.5x=3.5L$ $x=\frac{7L}{3}$ $\frac{dx}{dL}=\frac{7}{3}$ $\frac{dx}{dt}=2$ $\frac{dL}{dt}=\frac{dL}{dx}\times\frac{dx}{dt}$ $=\frac{3}{7}\times 2$ $=\frac{6}{7}=0.857\text{ m s}^{-1}$	$\frac{dx}{dL}$ correct.		Correct solution with correct derivatives. (Units not required.)

(e)	Depth of water = x $h = x + 20$ $V = \frac{1}{3}h^3 - \frac{1}{3}20^3$ $= \frac{1}{3}(x+20)^3 - \frac{1}{3}20^3$ $\frac{dV}{dx} = (x+20)^2$ $A = (x+20)^2$ $\frac{dA}{dx} = 2(x+20)$ $\frac{dV}{dt} = 3000$ $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dV} \times \frac{dV}{dt}$ $= 2(x+20) \times \frac{1}{(x+20)^2} \times 3000$ When $x = 15$ $\frac{dA}{dt} = 2 \times 35 \times \frac{1}{35^2} \times 3000 = 171.4 \text{ cm}^2 \text{ min}^{-1}$	Correct $\frac{dV}{dx}$ OR $\frac{dA}{dx}$	Correct $\frac{dV}{dx}$ AND $\frac{dA}{dx}$	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q3	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	$f'(x)=\frac{5}{2x-3}\times 2=\frac{10}{2x-3}$ $\frac{10}{2x-3}=4$ $8x-12=10$ $x=2.75$	Correct solution with correct derivative.		
(b)	$f'(x)=\frac{e^{3x}-x.3e^{3x}}{\left(e^{3x}\right)^2}$ $=\frac{1-3x}{e^{3x}}$ $f'(x)=0\Rightarrow x=\frac{1}{3}$	Correct solution with correct derivative.		
(c)	$\frac{dx}{dt}=-3\sin t$ $\frac{dy}{dt}=3\cos 3t$ $\frac{dy}{dx}=\frac{3\cos 3t}{-3\sin t}=\frac{-\cos 3t}{\sin t}$ At $t=\frac{\pi}{4}$, $\frac{dy}{dx}=\frac{-\cos\left(\frac{3\pi}{4}\right)}{\sin\left(\frac{\pi}{4}\right)}=1$ $\therefore$ Gradient of normal $=-1$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	Correct solution with correct derivatives.	
(d)(i) (ii)	$\frac{dx}{dt}=-\text{Ak}\sin \text{k}t+\text{Bk}\cos \text{k}t$ $\frac{d^2x}{dt^2}=-\text{Ak}^2\cos \text{k}t-\text{B}k^2\sin \text{k}t$ $=-\text{k}^2\left(\text{A}\cos \text{k}t+\text{B}\sin \text{k}t\right)$ $=-\text{k}^2x$ $x(0)=0\Rightarrow \text{A}\cos 0+\text{B}\sin 0=0$ $\text{A}=0$ $v(0)=2\text{k}\Rightarrow 2\text{k}=-\text{Ak}\sin(0)+\text{Bk}\cos(0)$ $\text{B}=2$	Correct $\frac{dx}{dt}$ Or $\frac{d^2x}{dt^2}$ Consistent with incorrect $\frac{dx}{dt}$	Parts (i) and (ii) both correct.	

(e)	 $\cos A = \frac{2}{f}$ $f = \frac{2}{\cos A}$ $\sin A = \frac{w}{5-f}$ $w = (5-f)\sin A$ $= \left(5 - \frac{2}{\cos A}\right)\sin A$ $= 5\sin A - 2\tan A$ $\frac{dw}{dA} = 5\cos A - 2\sec^2 A$ $\frac{dw}{dA} = 0 \Rightarrow 5\cos A - 2\sec^2 A = 0$ $5\cos^3 A - 2 = 0$ $\cos^3 A = \frac{2}{5}$ $A = 42.5°$ $w = 1.55$ m	Differentiate corretedly related but incorrect expression for w.	Correct $\frac{dw}{dA}$	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 12	13 – 18	19 – 24