Assessment Schedule – 2016

Calculus: Apply differentiation methods in solving problems (91578)

Evidence Statement

Q1	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$\frac{dy}{dx} = 1 + x^{-2} - 2x^{-3}$	Correct solution		
(b)	$\frac{dh}{dt} = \frac{3.2\pi}{25}\cos\left(\frac{4\pi}{25}t + \frac{\pi}{2}\right)$ $= 0.402\cos\left(\frac{36\pi}{25} + \frac{\pi}{2}\right)$ = 0.395 metres per hour	Correct solution with correct derivative		
(c)	$\frac{dx}{dt} = -4\sin 2t$ $\frac{dy}{dt} = 2\tan t \sec^2 t$ $\frac{dy}{dx} = \frac{2\tan t \sec^2 t}{-4\sin 2t}$ $= \frac{2\tan t}{-4\sin 2t \cos^2 t}$ At $t = \frac{\pi}{4}$, $\frac{dy}{dx} = \frac{2}{-4 \times \left(\frac{1}{\sqrt{2}}\right)^2}$ $= \frac{2}{-2} = -1$	Correct $\frac{dx}{dt}$ or $\frac{dy}{dt}$	Correct solution with correct derivatives.	
(d)	$y = \frac{1}{4}(x-2)^2$ $\frac{dy}{dx} = \frac{1}{2}(x-2)$ At Q (6, 4) $\frac{dy}{dx} = \frac{1}{2}(6-2) = 2$ $\therefore$ At P $\frac{dy}{dx} = \frac{-1}{2}$ $\frac{-1}{2} = \frac{1}{2}(x-2)$ $-1 = x - 2$ $x = 1$	At P $\frac{dy}{dx} = \frac{-1}{2}$	Correct solution with correct derivative.	

(e)	$f(x)=\mathrm{e}^{-(x-k)^2}$ $f'(x)=-2(x-k)\mathrm{e}^{-(x-k)^2}$ $f''(x)=-2\mathrm{e}^{-(x-k)^2}+4(x-k)^2\mathrm{e}^{-(x-k)^2}$ $=\mathrm{e}^{-(x-k)^2}\left[4(x-k)^2-2\right]$ $f''(x)=0\Rightarrow 4(x-k)^2-2=0$ $4(x-k)^2=2$ $(x-k)^2=\frac{1}{2}$ $(x-k)=\frac{\pm1}{\sqrt{2}}$ $x=k\pm\frac{1}{\sqrt{2}}$	Correct $f'(x)$	Correct $f''(x)$	Correct solutions with correct $f'(x)$ and $f''(x)$

NØ	**N1**	**N2**	**A3**	**A4**	**M5**	**M6**	**E7**	**E8**
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q2	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$f'(x) = \ln(3x-1) + x \cdot \frac{3}{3x-1}$	Correct derivative		
(b)	$y = (2x-1)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}(2x-1)^{\frac{-1}{2}}.2$ $= \frac{1}{\sqrt{2x-1}}$ At $x = 5$, $\frac{dy}{dx} = \frac{1}{3}$	Correct solution with correct derivative.		
(c)(i) (ii)	1: –1, 1 2: –2, –1, 1, 4 3: –4, 3, $x > 4$ 4: $1 < x < 4$ 1	2 correct answers.	3 correct answers.	
(d)	$\frac{dV}{dt} = 4800 \text{ cm}^3 \text{ s}^{-1}$ $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dr} = 4\pi r^2$ $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ $= \frac{4800}{4\pi r^2} = \frac{1200}{\pi r^2}$ $V = 288000\pi = \frac{4}{3}\pi r^3$ $288000 = \frac{4}{3}r^3$ $r^3 = 216000$ $r = 60 \text{ cm}$ $\therefore \frac{dr}{dt} = \frac{1200}{\pi \times 60^2} = 0.106 \text{ cm s}^{-1}$	Correct expression for $\frac{dr}{dt}$	Correct solution with correct $\frac{dr}{dt}$ – units not required.	

(e)	$Vol = \frac{1}{3}\pi r^2 h$ $h = 6 + s$ $s^2 + r^2 = 6^2$ $r^2 = 36 - s^2$ $\therefore V = \frac{1}{3}\pi(36 - s^2)(6 + s)$ $= \frac{1}{3}\pi(216 + 36s - 6s^2 - s^3)$ $\frac{dV}{ds} = \frac{1}{3}\pi(36 - 12s - 3s^2)$ Max volume when $\frac{dV}{ds} = 0$ $\Rightarrow 3s^2 + 12s - 36 = 0$ $s^2 + 4s - 12 = 0$ $(s+6)(s-2) = 0$ $s = -6$ or $s = 2$ $s = 2$		Correct expression for $\frac{dV}{ds}$	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q3	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$f'(x) = \frac{1}{4}(3x+2)^{\frac{-3}{4}} \cdot 3$	Correct derivative.		
(b)	$y = 6x - e^{3x}$ $\frac{dy}{dx} = 6 - 3e^{3x}$ Want $\frac{dy}{dx} = 0$ $3e^{3x} = 6$ $e^{3x} = 2$ $x = \frac{\ln 2}{3} = 0.231$	Correct solution with correct derivative.		
(c)	$Area = A(x) = x(x-6)^2$ $= x^3 - 12x^2 + 36x$ $A'(x) = 3x^2 - 24x + 36$ Max when $A'(x) = 0$ $3(x^2 - 8x + 12) = 0$ $3(x-6)(x-2) = 0$ Max when $x = 2$ Max Area = $2 \times 16 = 32$	Correct expression for $A'(x)$	Correct solution for maximum area with correct derivative.	
(d)	$y = \frac{e^x}{\sin x}$ $\frac{dy}{dx} = \frac{\sin x \cdot e^x - e^x.\cos x}{\sin^2 x}$ $= \frac{\sin x \cdot e^x}{\sin^2 x} - \frac{e^x.\cos x}{\sin^2 x}$ $= \frac{e^x}{\sin x} - \frac{e^x}{\sin x} \cdot \frac{\cos x}{\sin x}$ $= y - y \cdot \cot x$ $= y(1 - \cot x)$	Correct expression for $\frac{dy}{dx}$.	Correct proof with correct derivative.	

(e)	$\tan\alpha = \frac{15}{d}$ $\quad\quad \tan(\alpha+\theta) = \frac{20.4}{d}$ $\tan\theta = \tan((\alpha+\theta)-\alpha)$ $= \frac{\tan(\alpha+\theta)-\tan\alpha}{1-\tan(\alpha+\theta)\cdot\tan\alpha}$ $= \frac{\frac{20.4}{d}-\frac{15}{d}}{1+\frac{20.4\times15}{d^2}}$ $= \frac{\frac{5.4}{d}}{\frac{d^2+306}{d^2}}$ $= \frac{5.4d}{d^2+306}$ Max when $\frac{\mathrm{d}(\tan\theta)}{\mathrm{d}d} = 0$ $\frac{(d^2+306)\times5.4-5.4d\times2d}{(d^2+306)^2} = 0$ $5.4d^2+306\times5.4-10.8d^2 = 0$ $5.4d^2-306\times5.4 = 0$ $d^2 = 306$ $d = 17.5$ m		Correct expression for $\frac{\mathrm{d}(\tan\theta)}{\mathrm{d}d}$ or $\frac{\mathrm{d}\theta}{\mathrm{d}d}$	Correct solution – units not required.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0–7	8–12	13–18	19–24