Assessment Schedule – 2017

Calculus: Apply differentiation methods in solving problems (91578)

Evidence Statement

Q 1	Evidence	Achievement	Merit	Excellence
(a)	$\frac{1}{2}x^{\frac{-1}{2}}+2\sec^2(2x)$	Correct solution.		
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+2).2\mathrm{e}^{2x}-\mathrm{e}^{2x}}{(x+2)^2}$ At $x=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\times2-1}{4}=\frac{3}{4}$	Correct solution with correct derivative.		
(c)	$y=0.5(x-3)^2+2$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2\times0.5\times(x-3)$ $=x-3$ At $x=1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-2$ $\therefore$ For normal $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}$ Through (1, 4) $\therefore$ Eqn of normal $y=\frac{1}{2}x+3.5$ At point P: $\frac{1}{2}x+3.5=0.5(x-3)^2+2$ $x+7=(x-3)^2+4$ $x+7=x^2-6x+9+4$ $x^2-7x+6=0$ $(x-6)(x-1)=0$ At point P $x=6$	Correct expression for $\frac{dy}{dx}$ (i.e. correct derivative).	Correct solution with correct derivative.	
(d)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{2}(t+1)^{\frac{-1}{2}}=\frac{1}{2\sqrt{t+1}}$ $\frac{\mathrm{d}y}{\mathrm{d}t}=2\cos2t$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2\cos2t.2\sqrt{t+1}$ $=4\cos2t.\sqrt{t+1}$ At $t=0$ $\frac{dy}{dx}=4\cos0\times\sqrt{1}$ $=4$	$\frac{\mathrm{d}x}{\mathrm{d}t}$ or $\frac{\mathrm{d}y}{\mathrm{d}t}$ correct.	Correct solution with correct derivatives.	
(e)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2-1).a-(ax-b).2x}{(x^2-1)^2}$ At $x=3$, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\Rightarrow 8a-(3a-b)\times6=0$ $-10a+6b=0$ $5a=3b$	Correct derivative.	Correct derivative plus one of the two equations relating *a* and *b*.	Correct solution with correct derivative.

	The curve passes through (3,1) $\Rightarrow 1 = \frac{3a-b}{8}$ $8 = 3a - b$ $24 = 9a - 3b$ $= 9a - 5a$ $= 4a$ $\therefore a = 6 \quad \text{and} \quad b = 10$			

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q 2	Evidence	Achievement	Merit	Excellence
(a)	$\frac{dy}{dx}=10(x^2-4x)^4.(2x-4)$	Correct derivative.		
(b)	$P(w)=96\ln(w+1.25)-16w-12$ $\frac{dP}{dw}=\frac{96}{w+1.25}-16$ Maximum when $\frac{dP}{dw}=0$ $\frac{96}{w+1.25}-16=0$ $96=16(w+1.25)$ $76=16w$ $w=4.75$	Correct solution with correct derivative.		
(c)	$y=\sqrt{x}$ $\frac{dy}{dx}=\frac{1}{2}x^{\frac{-1}{2}}=\frac{1}{2\sqrt{x}}$ At (4, 2) $\frac{dy}{dx}=\frac{1}{2\sqrt{4}}=\frac{1}{4}$ Tangent: $y=\frac{1}{4}x+c$ through (4,2) $2=1+c$ $c=1$ $y=\frac{1}{4}x+1$ $y=0 \Rightarrow 0=\frac{1}{4}x+1$ $x=-4$ Point Q is (–4,0).	Correct derivative.	Correct solution with correct derivative. Accept $x = -4$.	
(d)	$d^2=(4-x)^2+(\sqrt{x})^2$ $=16-8x+x^2+x$ $=16-7x+x^2$ Minimum distance $\Rightarrow \frac{d(d^2)}{dx}=0$ $\frac{d(d^2)}{dx}=-7+2x=0$ $x=3.5$ $y=\sqrt{x}=\sqrt{3.5}$ $P=(3.5,\sqrt{3.5})$	Correct expression for $\frac{dd}{dx}$ or $\frac{d(d^2)}{dx}$	Correct solution with correct derivative.	

	Alternative: $d=\left(x^2-7x+16\right)^{\frac{1}{2}}$ $\frac{\mathrm{d}d}{\mathrm{d}x}=\frac{1}{2}\left(x^2-7x+16\right)^{\frac{-1}{2}}.(2x-7)$ $=\frac{2x-7}{2\sqrt{x^2-7x+16}}$ Minimum when $\frac{\mathrm{d}d}{\mathrm{d}x}=0$ $2x-7=0$ etc			
(e)	$\text{Area}=2x\sqrt{r^2-x^2}$ $A(x)=2x\left(r^2-x^2\right)^{\frac{1}{2}}$ $A'(x)=2\left(r^2-x^2\right)^{\frac{1}{2}}+2x.\frac{1}{2}\left(r^2-x^2\right)^{\frac{-1}{2}}.(-2x)$ $=2\sqrt{r^2-x^2}-\frac{2x^2}{\sqrt{r^2-x^2}}$ $A'(x)=0\Rightarrow\sqrt{r^2-x^2}=\frac{x^2}{\sqrt{r^2-x^2}}$ $r^2-x^2=x^2$ $2x^2=r^2$ $x^2=\frac{r^2}{2}$ $x=\frac{r}{\sqrt{2}}$		Correct derivative.	Correct solution presented in a correct mathematical manner.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q 3	Evidence	Achievement	Merit	Excellence
(a)	$\frac{dy}{dx} = x.\frac{3}{3x-1} + \ln(3x-1)$	Correct derivative.		
(b)	$y = x^{-1} - x^{-2}$ $\frac{dy}{dx} = -x^{-2} + 2x^{-3}$ $= \frac{-1}{x^2} + \frac{2}{x^3}$ At $x = 2$ $\frac{dy}{dx} = \frac{-1}{4} + \frac{2}{8} = 0$	Correct solution with correct derivative.		
(c)	(i) 1. $x < -2$, $x = 2$ 2. -2, 1 3. -1, 0 4. $x > 1$ (ii) 2	2 correct answers.	3 correct answers.	
(d)	Let h = height above Sarah's eye level. $\tan\theta = \frac{h}{30}$ $h = 30\tan\theta$ $\frac{dh}{d\theta} = 30\sec^2\theta$ $\frac{dh}{dt} = 2$ $\frac{d\theta}{dt} = \frac{dh}{dt} \times \frac{d\theta}{dh}$ $= 2 \times \frac{1}{30\sec^2\theta}$ $= \frac{\cos^2\theta}{15}$ *At* $h = 20$ $\theta = \tan^{-1}\left(\frac{20}{30}\right) = 0.588$ $\frac{d\theta}{dt} = \frac{(\cos 0.588)^2}{15}$ $= 0.046$ radians per second	Correct expression for $\frac{dh}{d\theta}$	Correct solution with correct derivatives. Ignore units in the solution.	

(e)	(i) $\frac{dy}{dx} = e^x.\cos kx + e^x(-k\sin kx)$ $= e^x(\cos kx - k\sin kx)$ $\frac{d^2y}{dx^2} = e^x(\cos kx - k\sin kx)$ $+e^x(-k\sin kx - k^2\cos kx)$ $= e^x(\cos kx - 2k\sin kx - k^2\cos kx)$ (ii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$ $\Rightarrow e^x(\cos kx - 2k\sin kx - k^2\cos kx)$ $-2e^x(\cos kx - k\sin kx) + 2e^x\cos kx = 0$ $\Rightarrow e^x(\cos kx - k^2\cos kx) = 0$ $e^x\cos kx(1-k^2) = 0$ $k = \pm 1$	Correct expression for $\frac{dy}{dx}$	Correct expression for $\frac{d^2y}{dx^2}$	Correct solution with correct derivatives.

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No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 13	14 – 19	20 – 24