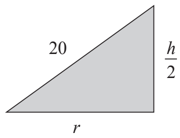
Assessment Schedule – 2020

Calculus: Apply differentiation methods in solving problems (91578)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\frac{dy}{dx} = 5\left(3x - x^2\right)^4 \cdot \left(3 - 2x\right)$	Correct derivative.		
(b)	$y = 3\sin 2x + \cos 2x$ $\frac{dy}{dx} = 6\cos 2x - 2\sin 2x$ At $x = \frac{\pi}{4}$ $\quad \frac{dy}{dx} = 6\cos\frac{\pi}{2} - 2\sin\frac{\pi}{2} = -2$	Correct gradient with correct derivative.		
(c)	$\frac{dy}{dx} = \frac{\left(1+\ln x\right)\cdot 1 - x \cdot \frac{1}{x}}{\left(1+\ln x\right)^2}$ $= \frac{\ln x}{\left(1+\ln x\right)^2}$ $\frac{dy}{dx} = 0 \Rightarrow \ln x = 0$ $x = 1$	Correct derivative.	Correct solution with correct derivative.	
(d)	$\frac{dy}{dx} = x^2 \cdot -\sin x + 2x\cos x$ At $x = \pi$ $\quad \frac{dy}{dx} = \pi^2 \cdot \left(-\sin\pi\right) + 2\pi\cos\pi$ $= -2\pi$ At $x = \pi$ $\quad y = -\pi^2$ Tangent equation $y - y_1 = m\left(x - x_1\right)$ $y + \pi^2 = -2\pi\left(x - \pi\right)$ $y + \pi^2 = -2\pi x + 2\pi^2$ $y + 2\pi x = \pi^2$	Correct derivative.	Correct proof with correct derivative.	

	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(e)	$r^2 + \left(\frac{h}{2}\right)^2 = 400$ $r^2 = 400 - \frac{h^2}{4}$  $V_{\text{cyl}} = \pi r^2 h$ $= \pi\left(400 - \frac{h^2}{4}\right)h$ $= \pi\left(400h - \frac{h^3}{4}\right)$ $\frac{dV}{dh} = \pi\left(400 - \frac{3h^2}{4}\right)$ $\frac{dV}{dh} = 0 \Rightarrow 400 - \frac{3h^2}{4} = 0$ $h = \sqrt{\frac{1600}{3}} = \frac{40}{\sqrt{3}} = 23.1\text{ cm}$ $r = 16.3\text{ cm}$ $V = \pi \times 16.3^2 \times 23.1$ $= 19\,300\text{ cm}^3$ $V = 19\,347\text{ cm}^3$	Correct expression for $\frac{dV}{dh}$ or $\frac{dV}{dr}$	Correct value of r or h with correct derivatives. Units not required.	Correct solution with correct derivatives. Units not required.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with one minor error	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\frac{dy}{dx} = \frac{x^3 \cdot \sec^2 x - 3x^2 \tan x}{x^6}$	Correct derivative		
(b)	$\frac{dV}{dt} = -4250e^{-0.25t} - 1000e^{-0.5t}$ $t = 8 \Rightarrow \frac{dV}{dt} = -4250e^{-2} - 1000e^{-4}$ $= -593.50$ Decreasing at \$593.50 per year.	Correct solution with correct derivative. Units not required. Interpretation not required.		
(c)	$f'(x) = (2x-3)2xe^{x^2+k} + 2e^{x^2+k}$ $= e^{x^2+k}((2x-3)2x+2)$ $= e^{x^2+k}(4x^2-6x+2)$ $= 2e^{x^2+k}(2x^2-3x+1)$ $f'(x) = 0 \Rightarrow 2e^{x^2+k} = 0$ or $2x^2 - 3x + 1 = 0$ $2e^{x^2+k}$ has no solutions since $2e^{x^2+k}$ is always positive. $2x^2 - 3x + 1 = 0$ $(2x-1)(x-1) = 0$ $x = \frac{1}{2}$ or $x = 1$	Correct derivative.	Correct solution with correct derivative. Reference to $2e^{x^2+k} = 0$ is not required	
(d)	$\tan\theta = \frac{h}{500}$ $h = 500\tan\theta$ $\frac{dh}{d\theta} = 500\sec^2\theta = \frac{500}{\cos^2\theta}$ $t = 10$ $\tan\theta = \frac{480}{500}$ $\theta = 0.765$ $\frac{d\theta}{dt} = \frac{dh}{dt} \times \frac{d\theta}{dh}$ $= 9.6t \times \frac{\cos^2\theta}{500}$ $= 96 \times \frac{\cos^2(0.765)}{500}$ $= 0.0999$ (accept 0.1)	Correct expression for $\frac{dh}{d\theta}$.	Correct expression for $\frac{d\theta}{dt}$.	Correct solution with correct derivatives.

	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(e)	$\frac{dx}{dt} = \frac{1}{t} \quad \frac{dy}{dt} = 18t^2$ $\frac{dy}{dx} = 18t^3$ $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right).\frac{dt}{dx}$ $= 54t^2 \times t$ $= 54t^3$ $54t^3 = 2$ $t^3 = \frac{1}{27}$ $t = \frac{1}{3}$ $x = \ln\left(\frac{1}{3}\right)$ $y = 6\left(\frac{1}{3}\right)^3$ $= \frac{2}{9}$ P is $\left(\ln\left(\frac{1}{3}\right), \frac{2}{9}\right)$	Correct expression for $\frac{dy}{dx}$.	Correct expression for $\frac{d^2y}{dx^2}$.	Correct solution with correct derivatives. Accept (–1.1, 0.22).

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t	2t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3 \times \frac{1}{x^2-1} \times 2x$ $= \frac{6x}{x^2-1}$	Correct derivative.		
(b)	$f(x) = 2x - 2\sqrt{x}$ $f'(x) = 2 - x^{\frac{-1}{2}}$ $\Rightarrow 2 - \frac{1}{\sqrt{x}} = 1$ $\frac{1}{\sqrt{x}} = 1$ $x = 1$	Correct value of x with correct derivative		
(c)	$y = (2x+1)^{\frac{1}{2}}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}(2x+1)^{\frac{-1}{2}} \times 2$ $= (2x+1)^{\frac{-1}{2}}$ $= \frac{1}{\sqrt{2x+1}}$ At $x = 4$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}$ Normal gradient $= -3$ $y - 3 = -3(x-4)$ $y = -3x + 15$ x-intercept $\Rightarrow y = 0$ $x = 5$	Correct derivative.	Correct solution with correct derivative. Must have correct gradient of normal to justify $x = 5$	
(d)	$y = (x-3)^{-1} + x$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -1(x-3)^{-2} + 1$ $= \frac{-1}{(x-3)^2} + 1$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow x - 3 = \pm 1$ $x = 2$ or 4 $\frac{\mathrm{d}^2x}{\mathrm{d}y^2} = \frac{2}{(x-3)^3}$ $x = 2 \Rightarrow \frac{\mathrm{d}^2x}{\mathrm{d}y^2} < 0$ Local max at $x = 2$ $x = 4 \Rightarrow \frac{\mathrm{d}^2x}{\mathrm{d}y^2} > 0$ Local min at $x = 4$	Correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.	Correct expressions for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ OR Correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ plus x-coordinates of TPs found and nature stated without correct use of first or second derivative test.	Correct solution with correct derivatives. With use of the first derivative test or second derivative test to justify the nature of the turning points.

	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(e)	$\frac{dy}{dx} = (3x+2)e^{-2x}.(-2) + 3e^{-2x}$ $= e^{-2x}[-2(3x+2)+3]$ $= e^{-2x}(-6x-1)$ $\frac{d^2y}{dx^2} = -6e^{-2x} - 2e^{-2x}(-6x-1)$ $= e^{-2x}[-6-2(-6x-1)]$ $= e^{-2x}(-6+12x+2)$ $= e^{-2x}(12x-4)$ $= 4e^{-2x}(3x-1)$ EITHER $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ LHS$= 4e^{-2x}(3x-1) + 4e^{-2x}(-6x-1) + 4e^{-2x}(3x+2)$ $= 4e^{-2x}[3x-1-6x-1+3x+2]$ $= 0$ = RHS as required OR LHS$= e^{-2x}(12x-4) + 4e^{-2x}(-6x-1) + 4e^{-2x}(3x+2)$ $= e^{-2x}[12x-4+4(-6x-1)+4(3x+2)]$ $= e^{-2x}[12x-4+24x-4+12x+8]$ $= 0$ = RHS as required	Correct expression for $\frac{dy}{dx}$.	Correct expression for $\frac{d^2y}{dx^2}$.	Correct solution with correct derivatives.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t	2t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 8	9 – 14	15 – 20	21 – 24