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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 3 Calculus, 2016

91578 Apply differentiation methods in solving problems

9.30 a.m. Wednesday 23 November 2016
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

(a) Differentiate $y = 1 + x - \frac{1}{x} + \frac{1}{x^2}$.

(b) The height of the tide at a particular beach today is given by the function

$$h(t) = 0.8\sin\left(\frac{4\pi}{25}t + \frac{\pi}{2}\right)$$

where h is the height of water, in metres, relative to the mean sea level and t is the time in hours after midnight.

c2kiwi.blogspot.co.nz/2011/01/christchurch-wedding-stroll-on-beach.html

At what rate was the height of the tide changing at that beach at 9.00 a.m. today?

(c) A curve is defined by the parametric equations

$x = 2\cos 2t$ and $y = \tan^2 t$.

Find the gradient of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

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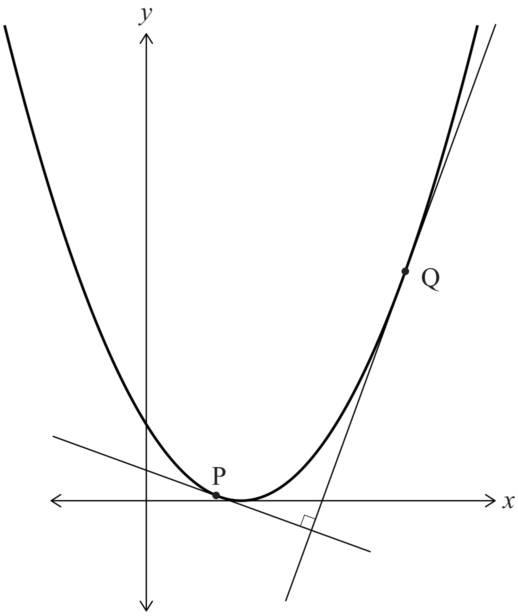
(d) The tangents to the curve $y = \frac{1}{4}(x-2)^2$ at points P and Q are perpendicular.



Q is the point $(6, 4)$.

What is the x-coordinate of point P?

You must use calculus and show any derivatives that you need to find when solving this problem.

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(e) A curve is defined by the function $f(x) = e^{-(x-k)^2}$.

Find, in terms of k, the x-coordinate(s) for which $f''(x) = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

QUESTION TWO

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(a) Differentiate $f(x) = x\ln(3x - 1)$.

(b) Find the gradient of the tangent to the function $y = \sqrt{2x-1}$ at the point $(5, 3)$.

You must use calculus and show any derivatives that you need to find when solving this problem.

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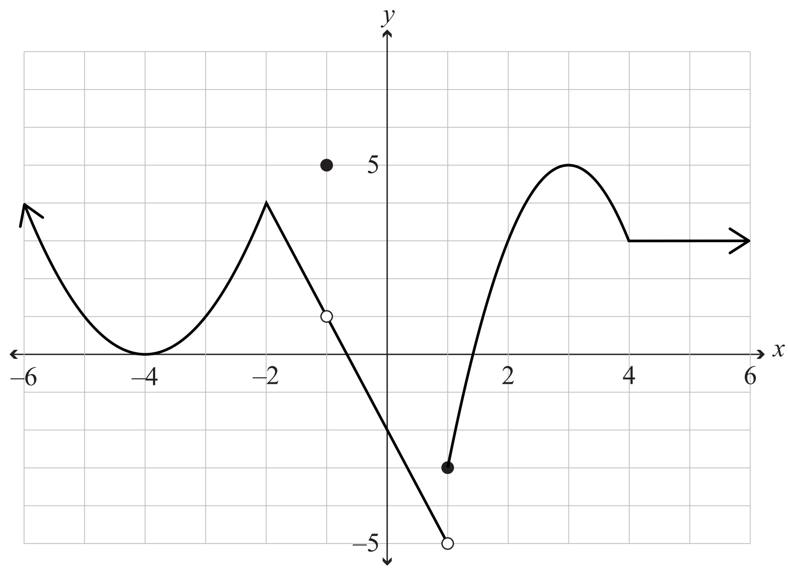
(c) The graph below shows the function $y = f(x)$.



For the function $y = f(x)$ above:

(i) Find the value(s) of x that meet the following conditions:

1. f is not continuous: ______

2. f is not differentiable: ______

3. $f'(x) = 0$: ______

4. $f''(x) < 0$: ______

(ii) What is the value of $\lim_{x \to -1} f(x)$?

State clearly if the value of the limit does not exist.

______

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(d) A large spherical helium balloon is being inflated at a constant rate of 4800 $cm^3\ s^{-1}$.

At what rate is the radius of the balloon increasing when the volume of the balloon is $288\,000\pi$ cm^3?

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) A cone of height h and radius r is inscribed, as shown, inside a sphere of radius 6 cm.

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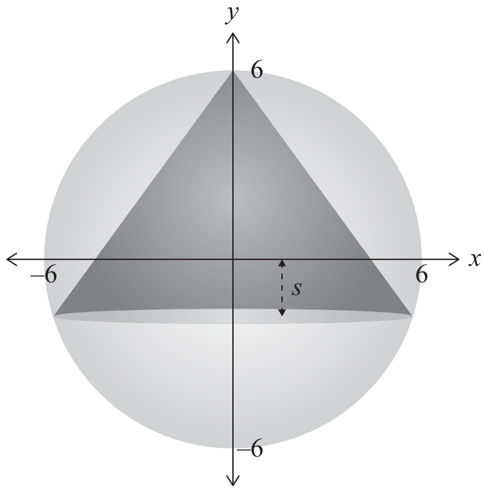

The base of the cone is s cm below the x-axis.

Find the value of s which maximises the volume of the cone.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the volume you have found is a maximum.

QUESTION THREE

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(a) Differentiate $f(x)=\sqrt[4]{3x+2}$.

(b) Find the x-value at which a tangent to the curve $y = 6x - e^{3x}$ is parallel to the x-axis.

You must use calculus and show any derivatives that you need to find when solving this problem.

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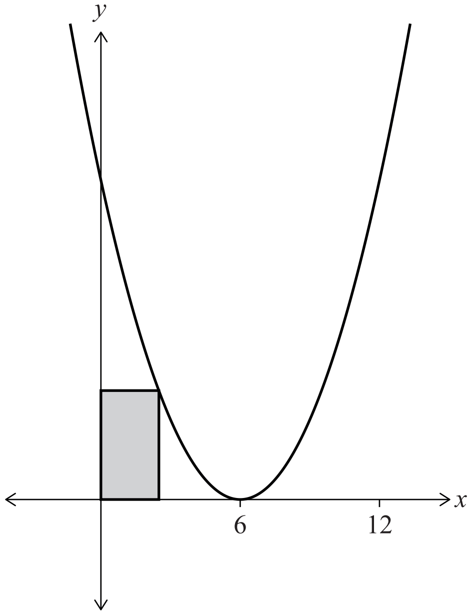
(c) A rectangle has one vertex at (0,0) and the opposite vertex on the curve $y = (x - 6)^2$, where $0 < x < 6$, as shown on the graph below.



Find the maximum possible area of the rectangle.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the area you have found is a maximum.

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(d) If $y = \dfrac{e^x}{\sin x}$, show that $\dfrac{dy}{dx} = y(1 - \cot x)$.

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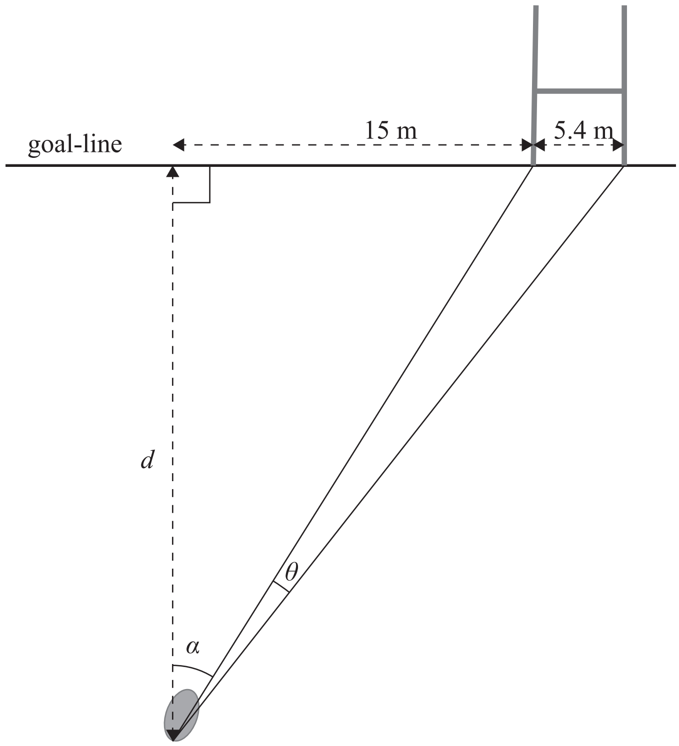
(e) In a rugby game, a try is scored 15 m from the left-hand goal-post. The conversion kick is taken at some point on the line perpendicular to the goal-line from the point where the try was scored, as shown in the diagram below.

The ball needs to pass between the goal-posts, which are 5.4 m apart.



Find the distance d from the goal-line that the conversion kick should be taken from in order to maximise the angle θ between the lines from the ball to the goal-posts.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not need to prove that the angle you have found is a maximum.

Extra paper if required.
Write the question number(s) if applicable.

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