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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 3 Calculus, 2017

91578 Apply differentiation methods in solving problems

9.30 a.m. Thursday 23 November 2017
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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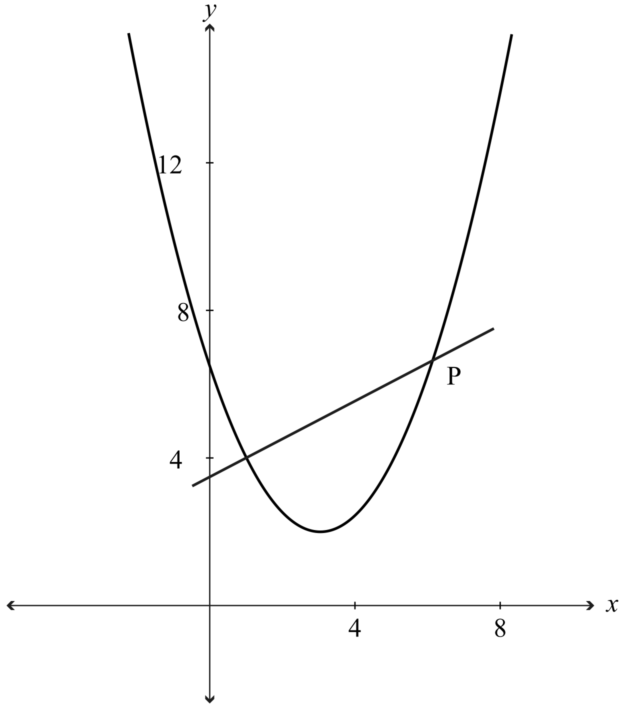
(a) Differentiate $y = \sqrt{x} + \tan(2x)$.

(b) Find the gradient of the tangent to the curve $y = \dfrac{e^{2x}}{x+2}$ at the point where $x = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) The normal to the parabola $y = 0.5(x - 3)^2 + 2$ at the point (1,4) intersects the parabola again at the point P.



Find the x-coordinate of point P.

You must use calculus and show any derivatives that you need to find when solving this problem.

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(d) A curve is defined parametrically by the equations $x = \sqrt{t+1}$ and $y = \sin 2t$.

Find the gradient of the tangent to the curve at the point when $t = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Find the values of a and b such that the curve $y = \dfrac{ax - b}{x^2 - 1}$ has a turning point at (3,1).

You must use calculus and show any derivatives that you need to find when solving this problem.

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QUESTION TWO

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(a) Differentiate $y = 2(x^2 - 4x)^5$.

You do not need to simplify your answer.

(b) The percentage of seeds germinating depends on the amount of water applied to the seedbed that the seeds are sown in, and may be modelled by the function:

$$P(w) = 96\ln(w + 1.25) - 16w - 12$$

where P is the percentage of seeds that germinate and
w is the daily amount of water applied (litres per square metre of seedbed), with $0 \leq w \leq 15$.

Find the amount of water that should be applied daily to maximise the percentage of seeds germinating.

You must use calculus and show any derivatives that you need to find when solving this problem.

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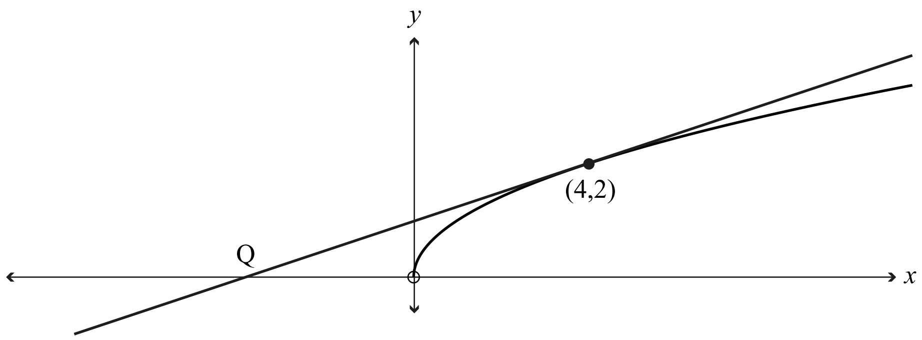
(c) The tangent to the curve $y = \sqrt{x}$ is drawn at the point (4,2).



Find the co-ordinates of the point Q where the tangent intersects the x-axis.

You must use calculus and show any derivatives that you need to find when solving this problem.

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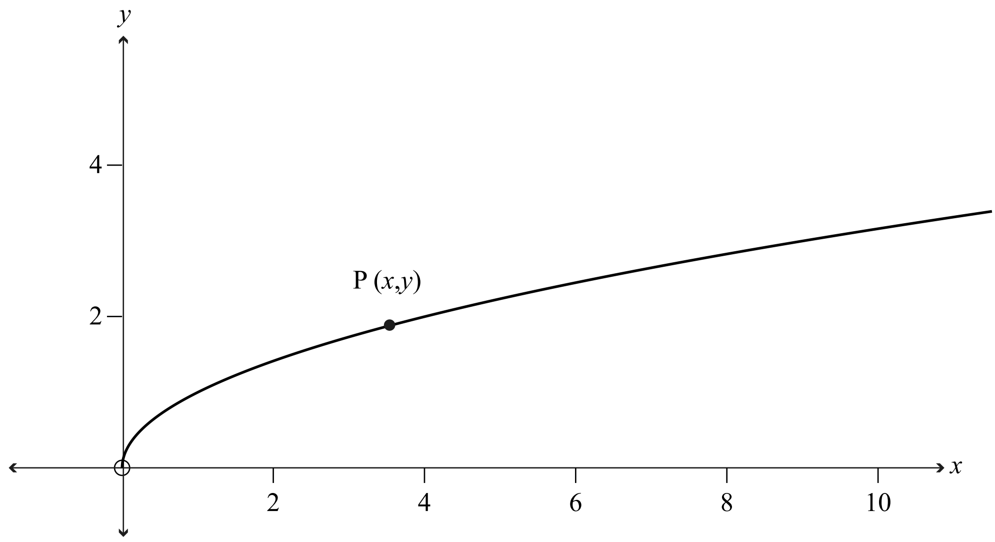
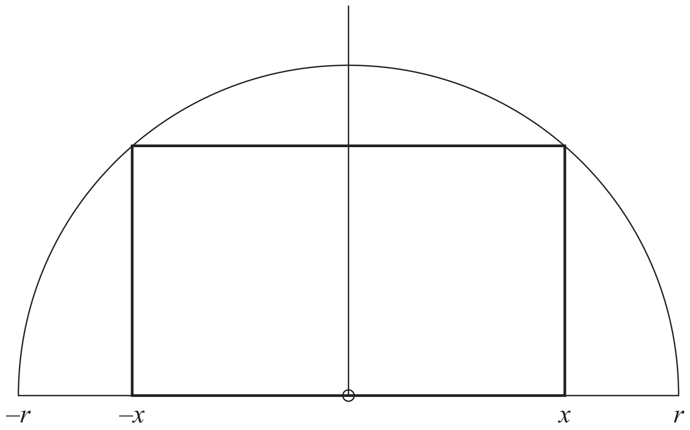
(d) Find the coordinates of the point P (x,y) on the curve $y = \sqrt{x}$ that is closest to the point (4,0).



You do not need to prove that your solution is the minimum value.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) A rectangle is inscribed in a semi-circle of radius r, as shown below.



Show that the maximum possible area of such a rectangle occurs when $x = \frac{r}{\sqrt{2}}$.

You do not need to prove that your solution gives the maximum area.

You must use calculus and show any derivatives that you need to find when solving this problem.

QUESTION THREE

ASSESSOR'S USE ONLY

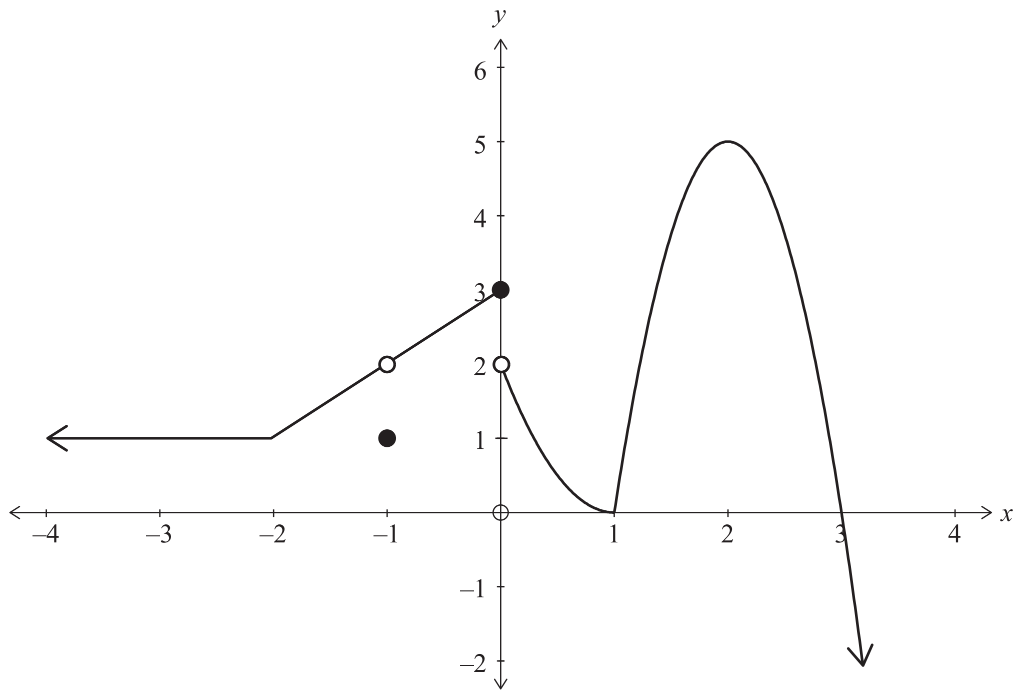
(a) Differentiate $y = x\ln(3x - 1)$.

You do not need to simplify your answer.

(b) Find the gradient of the curve $y = \frac{1}{x} - \frac{1}{x^2}$ at the point $\left(2, \frac{1}{4}\right)$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) The graph below shows the function $y = f(x)$.



For the function above:

(i) Find the value(s) of x that meet the following conditions:

(1) $f'(x) = 0$: ______

(2) $f(x)$ is continuous but not differentiable: ______

(3) $f(x)$ is not continuous: ______

(4) $f''(x) < 0$: ______

(ii) What is the value of $\lim_{x \to -1} f(x)$? ______

State clearly if the value does not exist.

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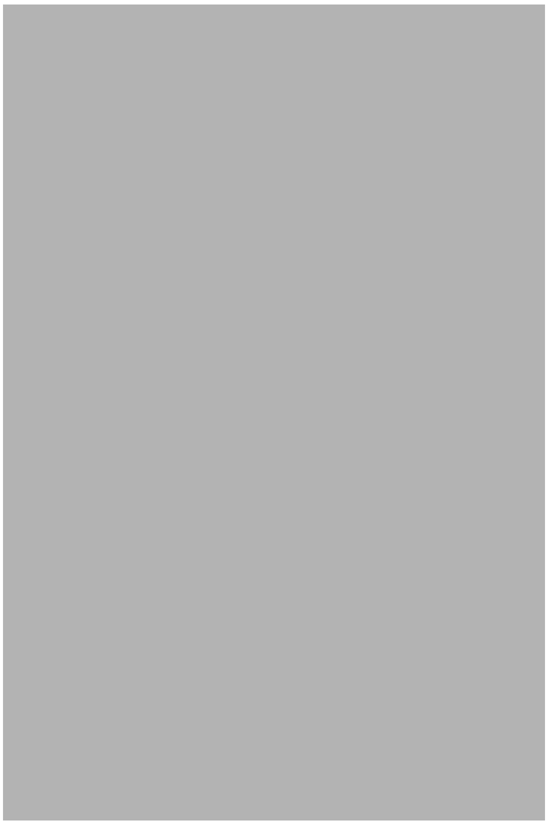
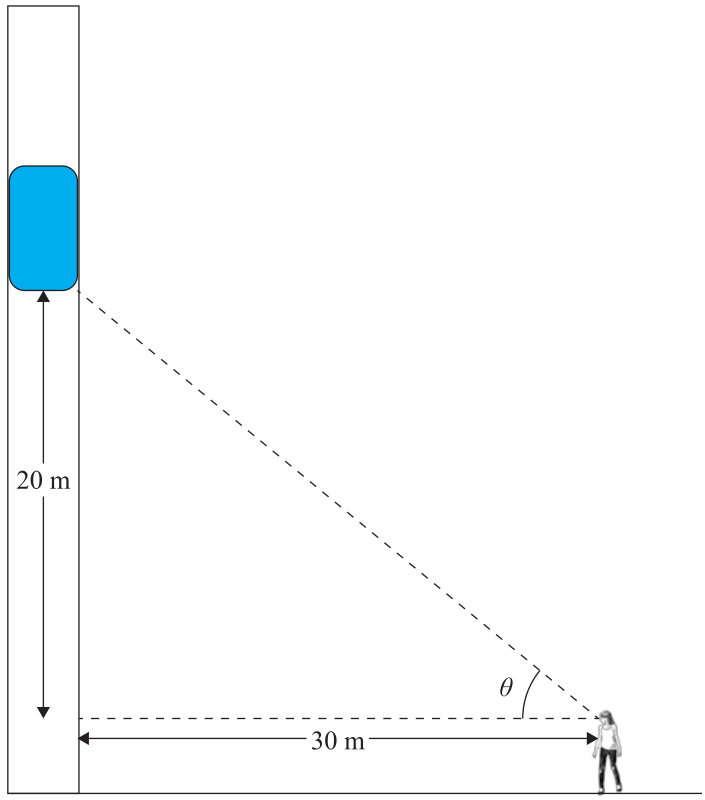
(d) A building has an external elevator. The elevator is rising at a constant rate of 2 m s^{-1}.

Sarah is stationary, watching the elevator from a point 30 m away from the base of the elevator shaft.

Let the angle of elevation of the elevator floor from Sarah's eye level be θ.



www.alibaba.com/product-detail/Sicher-external-elevator_60136882005.html



Find the rate at which the angle of elevation is increasing when the elevator floor is 20 m above Sarah's eye level.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) For the function $y = e^x \cos kx$:

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(ii) Find all the value(s) of k such that the function $y = e^x \cos kx$ satisfies the equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0 \text{ for all values of } x.$$

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Extra paper if required.
Write the question number(s) if applicable.

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