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SUPERVISOR'S USE ONLY

Level 3 Calculus, 2018

91578 Apply differentiation methods in solving problems

9.30 a.m. Tuesday 13 November 2018
Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

(a) Differentiate $y = 2x^3 + \dfrac{5}{\left(x^3+2\right)^3}$

You do not need to simplify your answer.

(b) If $f(x) = 3\cos 3x$, show that $9f(x) + f''(x) = 0$.

(c) Find the gradient of the curve $y = \ln\left|\sin^2 x\right|$ at the point where $x = \dfrac{\pi}{6}$

You must use calculus and show any derivatives that you need to find when solving this problem.

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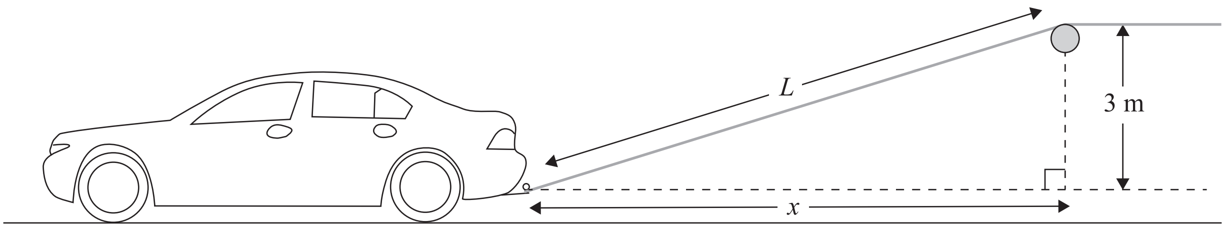
(d)



A car is being pulled along by a rope attached to the tow-bar at the back of the car.

The rope passes through a pulley, the top of which is 3 m further from the ground than the tow-bar.

The pulley is x m horizontally from the tow-bar, as shown in the diagram above.

The rope is being winched in at a speed of 0.6 m s^{-1}.

The wheels of the car remain in contact with the ground.

At what speed is the car moving when the length of the rope, L, between the tow-bar and the pulley is 5.4 m?

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) A curve is defined by the parametric equations $x = t^3 + 1$
$y = t^2 + 1$

Show that $\dfrac{\dfrac{d^2y}{dx^2}}{\left(\dfrac{dy}{dx}\right)^4}$ is a constant.

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QUESTION TWO

(a) Differentiate $y = 3\sqrt{x} + \operatorname{cosec} 5x$.

(b) A particle is travelling in a straight line. The distance, in metres, travelled by the particle may be modelled by the function

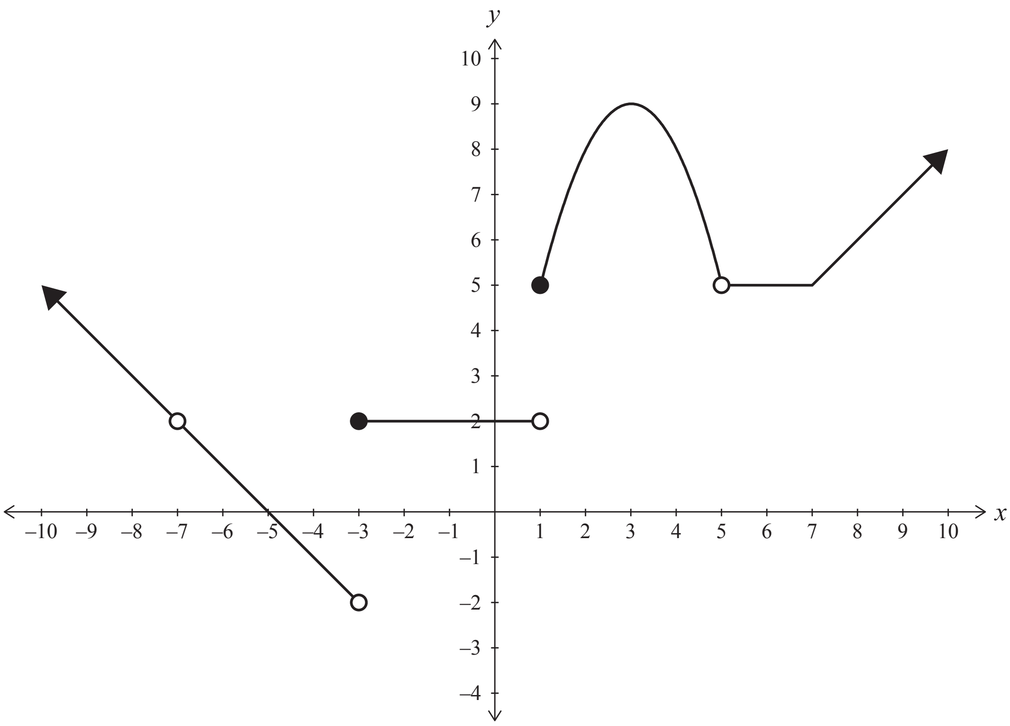
$$s(t) = \ln(3t^2 + 3t + 1) \qquad t \geq 0$$

where t is time measured in seconds.

Find the velocity of this particle after 2 seconds.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) The diagram below shows the graph of the function $y = f(x)$.



For the function above:

(i) What is the value of $f(1)$? ____________________

State clearly if the value does not exist.

(ii) For what value(s) of x does the function $f(x)$ not have a limit? ____________________

(iii) Find all the value(s) of x that meet the following conditions:

(1) $f'(x) > 0$: ____________________

(2) $f'(x) = 0$ and $f''(x) < 0$: ____________________

(3) $f(x)$ is continuous but not differentiable: ____________________

(d) If $y = e^x(2x^2 - x - 1)$, find the value(s) of x for which $\frac{dy}{dx} = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

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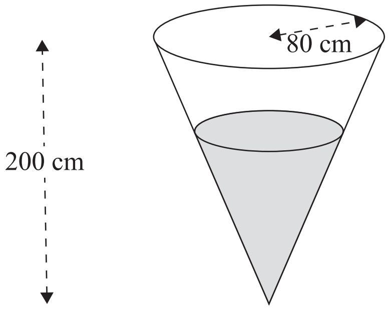
(e) A water tank is in the shape of an inverted right-circular cone.

The height of the cone is 200 cm and the radius of the cone is 80 cm.



The tank is being filled with water at a rate of 150 cm^3 per second.

At what rate will the surface area of the water in the tank be increasing when the depth of water in the tank is 125 cm?

You must use calculus and show any derivatives that you need to find when solving this problem.

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QUESTION THREE

(a) Differentiate $y = \dfrac{e^{2x}}{x^2+1}$.

You do not need to simplify your answer.

(b) A curve is defined parametrically by the parametric equations

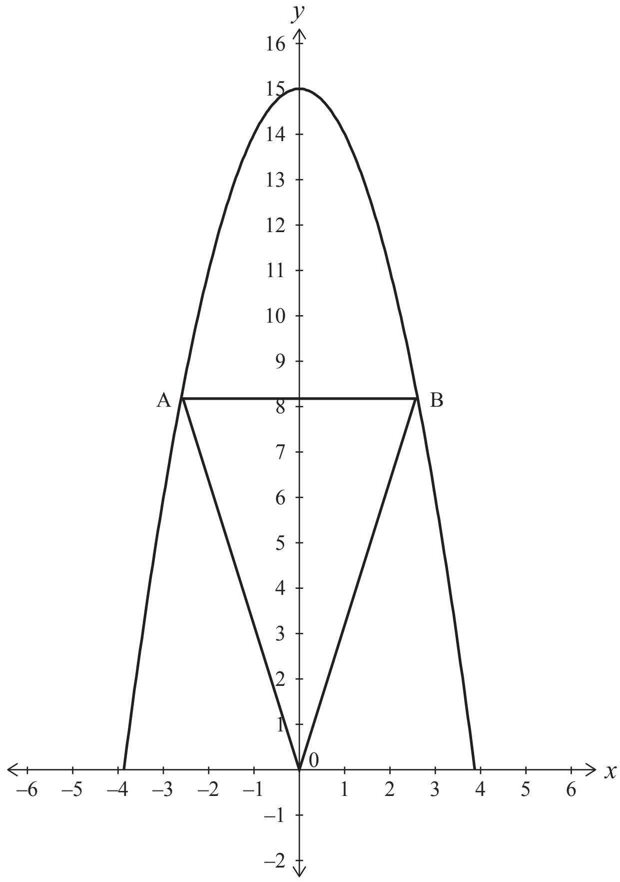
$x = 5e^{2t}$

$y = 2e^{5t}$

Find the gradient of the tangent to this curve at the point where $t = 0$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) The diagram below shows the graph of the function $y = 15 - x^2$, inside which an isosceles triangle OAB has been drawn.



Find the maximum possible area, A, of the triangle.

You may assume that your answer is a maximum.

You must use calculus and show any derivatives that you need to find when solving this problem.

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(d) Find the equation of the tangent to the curve $y = x^2 \ln x$ at the point where $x = \text{e}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on the following page.

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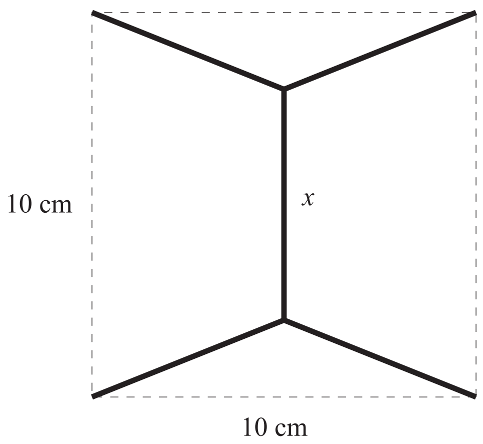
(e)



The above shape is made from wire. It has both vertical and horizontal lines of symmetry.

The ends of the shape are at the vertices of a square with a side length of 10 cm, as shown in the diagram above.

The length of the piece of wire through the centre of the shape is x cm.

Find the value(s) of x that enables the shape to be made with the minimum length of wire.

You do not need to prove that the length is a minimum.

You must use calculus and show any derivatives that you need to find when solving this problem.

Extra paper if required.
Write the question number(s) if applicable.

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