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NEW ZEALAND QUALIFICATIONS AUTHORITY
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QUALIFY FOR THE FUTURE WORLD
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Tick this box if you have NOT written in this booklet

Level 3 Calculus 2021

91578 Apply differentiation methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area ( ). This area may be cut off when the booklet is marked.

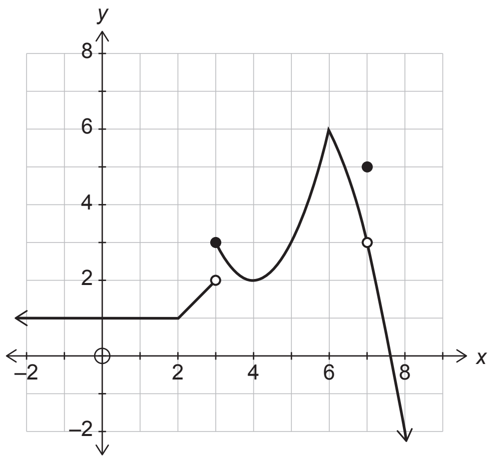
YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

(a) Differentiate $y = e^{3x}\sin 2x$.

You do not need to simplify your answer.

(b) The graph below shows the function $y = f(x)$.



For the function above:

(i) Find the value(s) of x that meet the following conditions:

(1) $f'(x) = 0$:

(2) $f(x)$ is concave upwards:

(ii) What is the value of $\lim_{x \to 7} f(x)$:

State clearly if the value does not exist.

(c) A curve has the equation $y = (2x+3)e^{x^2}$.

Find the x-coordinate(s) of any stationary point(s) on the curve.

You must use calculus and show any derivatives that you need to find when solving this problem.

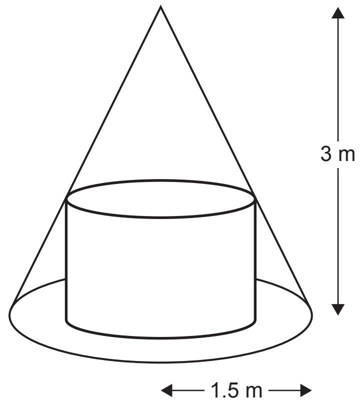
(d) A curve is defined parametrically by the equations $x = t^2 + 3t$ and $y = t^2 \ln(2t-3)$, for $t > \frac{3}{2}$.

Find the gradient of the tangent to the curve at the point (10,0).

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) A cone has a height of 3 m and a radius of 1.5 m.

A cylinder is inscribed in the cone, as shown in the diagram below.



The base of the cylinder has the same centre as the base of the cone.

Prove that the maximum volume of the cylinder is π m^3.

You must use calculus and show any derivatives that you need to find when solving this problem.

QUESTION TWO

(a) Differentiate $f(x)=\left(1-x^2\right)^5$.

You do not need to simplify your answer.

(b) A curve has the equation $y=\dfrac{x^2}{x+1}$.

Find the x-coordinate(s) of any stationary point(s) on the curve.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) A curve has the equation $y = (x^2 + 3x + 2)\cos 3x$.

Find the equation of the normal to the curve at the point where the curve crosses the y-axis.

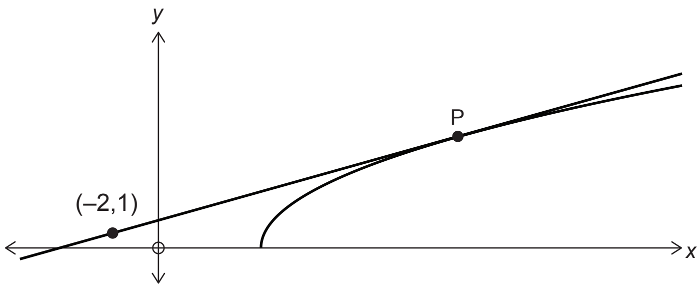
You must use calculus and show any derivatives that you need to find when solving this problem.

(d) The volume of a spherical balloon is increasing at a constant rate of 60 cm^3 per second.

Find the rate of increase of the radius when the radius is 15 cm.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) The graph below shows the curve $y = \sqrt{2x - 4}$, and the tangent to the curve at point P.
The tangent passes through the point (–2,1).



Find the coordinates of point P.

You must use calculus and show any derivatives that you need to find when solving this problem.

QUESTION THREE

(a) Differentiate $y = \dfrac{\cot x}{x^2 + 1}$.

You do not need to simplify your answer.

(b) The graph of the function $y = 4\sqrt{x} - x + 2$, where $x > 0$, has a stationary point at point Q.

Find the coordinates of point Q.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) For what values of x is the function $y = \dfrac{x}{x^2 + 4}$ increasing?

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) A curve has the equation $y = \dfrac{4x+k}{4x-k}$, where k is a constant and $x \neq \dfrac{k}{4}$.

The point P lies on the curve and has an x-coordinate of 3.

The gradient of the tangent to the curve at P is $\dfrac{-8}{27}$.

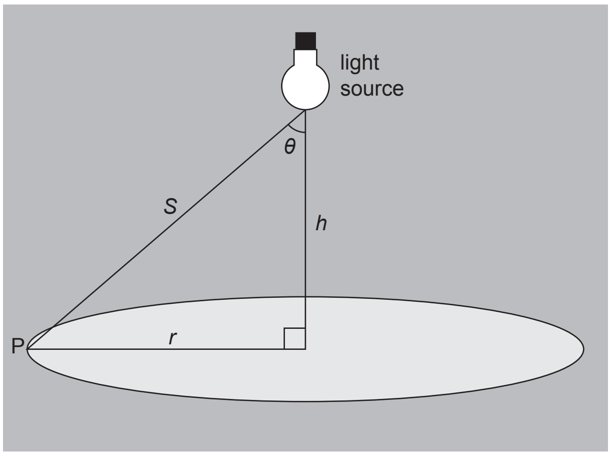
Find the possible value(s) of k.

You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on the next page.

(e) A lamp is suspended above the centre of a round table of radius r.

The height, h, of the lamp above the table is adjustable.



Point P is on the edge of the table.

At point P the illumination I is directly proportional to the cosine of angle θ in the above diagram, and inversely proportional to the square of the distance, S, to the lamp.

i.e. $I = \dfrac{k\cos\theta}{S^2}$, where k is a constant.

Prove that the edge of the table will have maximum illumination when $h = \dfrac{r}{\sqrt{2}}$.

You do not need to prove that your solution gives the maximum value.

You must use calculus and show any derivatives that you need to find when solving this problem.

Extra space if required.
Write the question number(s) if applicable.

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