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Level 3 Calculus 2022

91578 Apply differentiation methods in solving problems

Credits: Six

Achievement	Achievement with Merit	Achievement with Excellence
Apply differentiation methods in solving problems.	Apply differentiation methods, using relational thinking, in solving problems.	Apply differentiation methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–CALCF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

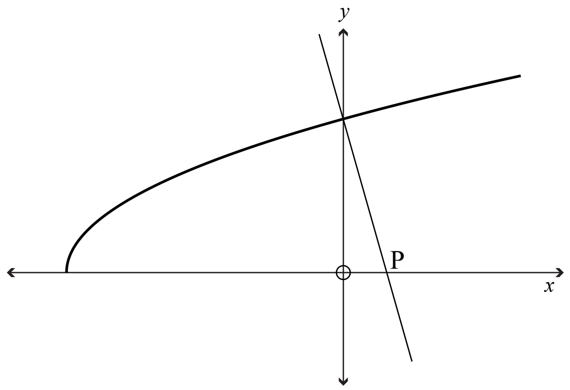
(a) Differentiate $y = (\ln x)^2$.

You do not need to simplify your answer.

(b) Find the x-value(s) of any stationary points on the graph of the function $f(x) = \dfrac{x^2 + 1}{x}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(c) The graph below shows the function $y=\sqrt{x+2}$, and the normal to the function at the point where the function intersects the y-axis.



Find the coordinates of point P, the x-intercept of the normal.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) A curve is defined parametrically by the equations:

$$x = 2 + 3t \text{ and } y = 3t - \ln(3t - 1) \text{ where } t > \frac{1}{3}.$$

Find the coordinates, (x,y), of any point(s) on the curve where the tangent to the curve has a gradient of $\frac{1}{2}$.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) If p is a positive real constant, prove that $y = e^{px^2}$ does not have any points of inflection.

You must use calculus and show any derivatives that you need to find when solving this problem.

QUESTION TWO

(a) Differentiate $f(x) = (5x - 3)\sin(4x)$.

You do not need to simplify your answer.

(b) Find the gradient of the tangent to the curve $y = \left(3x^2 - 2\right)^3$ when $x = 2$.

You must use calculus and show any derivatives that you need to find when solving this problem.

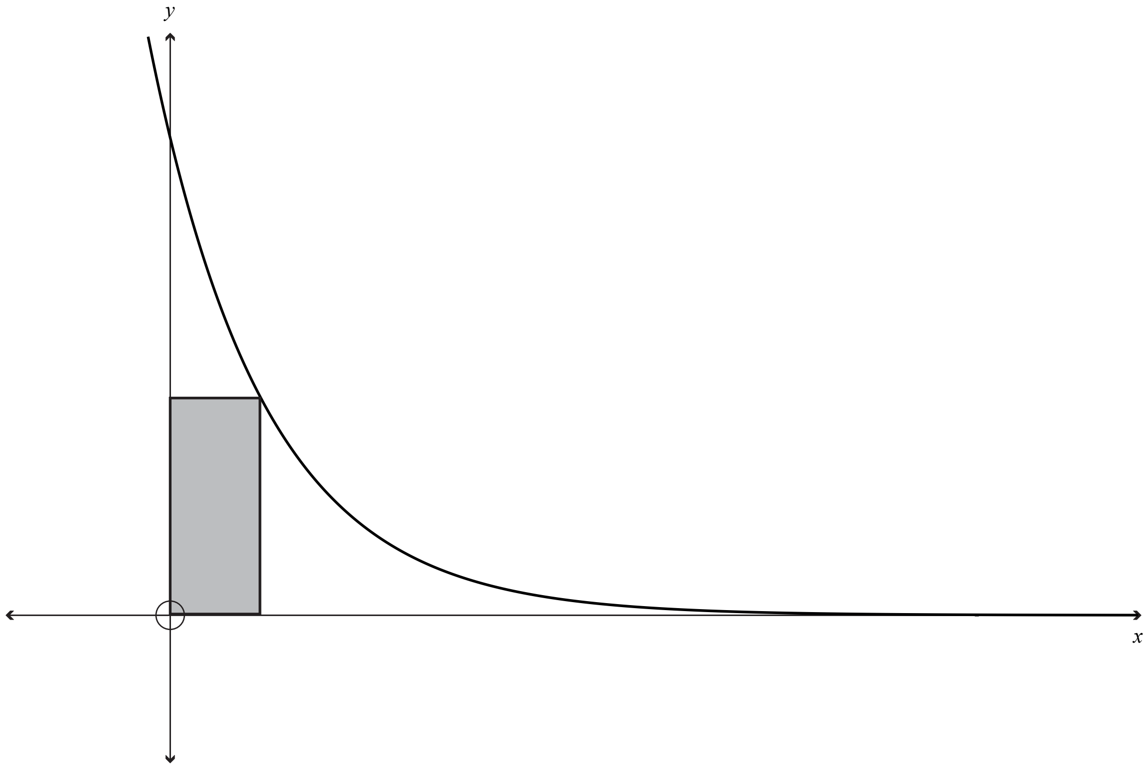
(c) An object is travelling in a straight line. Its displacement, in metres, is given by the formula:

$d(t) = \dfrac{t^2 - 6}{2t^3}$ where $t > 0$, t is time in seconds.

Find the time(s) when the object is stationary.

You must use calculus and show any derivatives that you need to find when solving this problem.

(d) A rectangle has one vertex at (0,0) and the opposite vertex on the curve $y = 6e^{1-0.5x}$, where $x > 0$, as shown on the graph below.

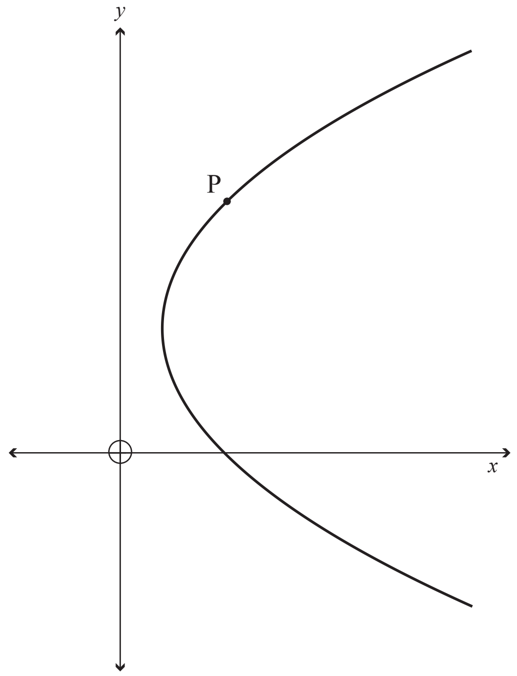


Find the maximum possible area of the rectangle.

You must use calculus and show any derivatives that you need to find when solving this problem.

You do not have to prove that the area you have found is a maximum.

(e) The curve with the equation $(y-5)^2 = 16(x-2)$ has a tangent of gradient 1 at point P.



This tangent intersects the x and y axes at points R and S respectively.

Prove that the length RS is $7\sqrt{2}$.

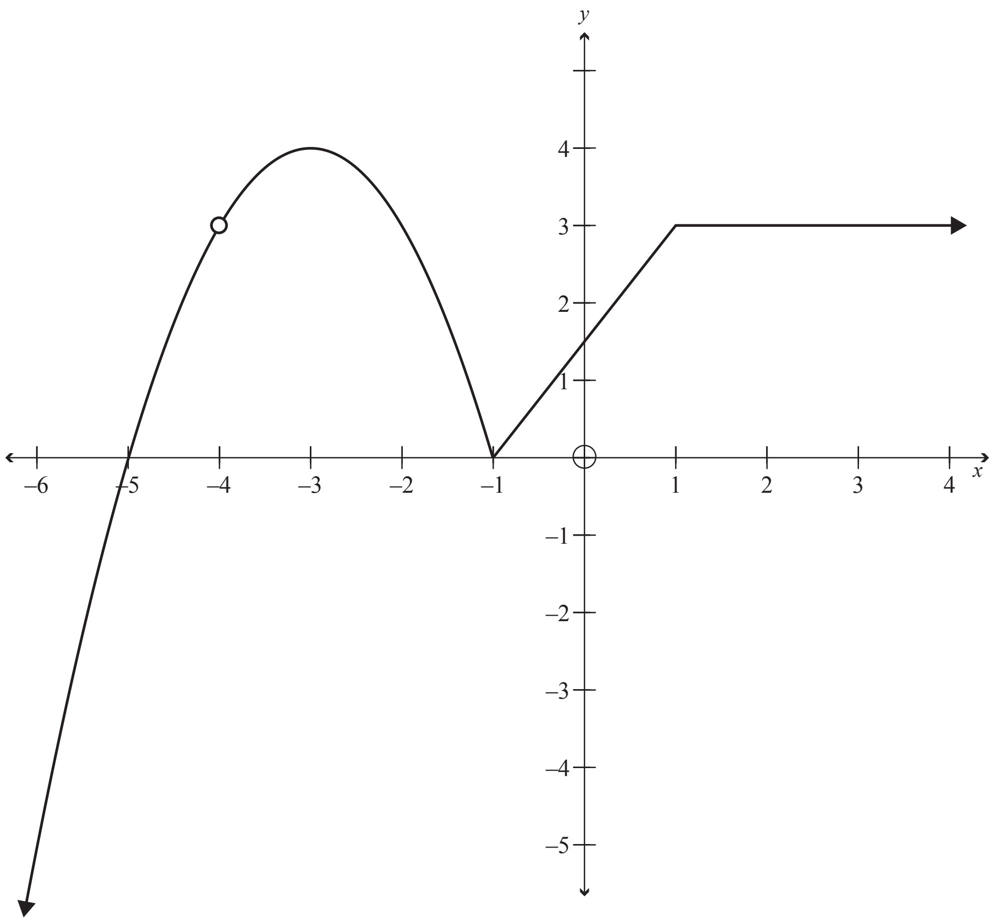
You must use calculus and show any derivatives that you need to find when solving this problem.

QUESTION THREE

(a) Differentiate $y = e^{4\sqrt{x}}$.

You do not need to simplify your answer.

(b) The graph below shows the function $y = f(x)$.



For the function above:

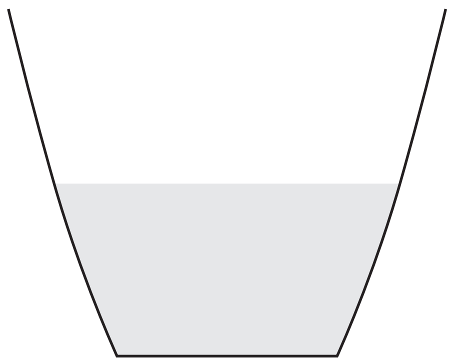
(i) Find the value(s) of x where $f(x)$ is not differentiable.

(ii) Find the value(s) of x for which $f'(x) = 0$.

(iii) What is the value of $\lim_{x \to -4} f(x)$?

(State clearly if the value does not exist.)

(c) The diagram below shows the cross-section of a bowl containing water.



When the height of the water level in the bowl is h cm, the volume, V cm^3, of water in the bowl is given by $V = \pi\left(\frac{3}{2}h^2 + 3h\right)$.

Water is poured into the bowl at a constant rate of 20 cm^3 s^{-1}.

Find the rate, in cm s^{-1}, at which the height of the water level is increasing when the height of the water level is 3 cm.

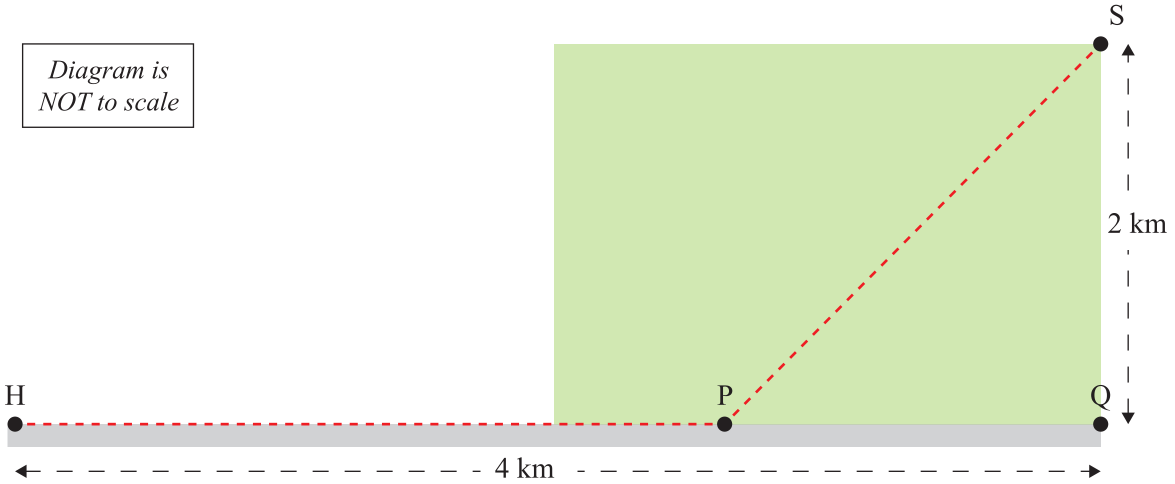
You must use calculus and show any derivatives that you need to find when solving this problem.

Question Three continues on the next page.

(d) Find the x-value(s) of any stationary point(s) on the graph of the function $y = 9x - 2 + \frac{3}{3x-1}$ and determine their nature.

You must use calculus and show any derivatives that you need to find when solving this problem.

(e) Megan cycles from her home, H, to school, S, each day.



She rides along a path from her home to point P at a constant speed of 10 kilometres per hour.

At point P, Megan cuts across a park, heading directly to school. When cycling across the park, Megan can only cycle at 6 kilometres per hour.

At what distance from her home should she choose to cut across the park in order to make her travelling time a minimum?

You must use calculus and show any derivatives that you need to find when solving this problem.

Extra space if required.
Write the question number(s) if applicable.

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