## Assessment Schedule – 2013

## Mathematics and Statistics (Statistics): Apply probability distributions in solving problems (91586)

## Evidence Statement

One	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	Normal distribution, $\mu = 44.6$ , $\sigma = 7.3$ P(X < 40) + P(X > 50) = 0.2643 + 0.2297 = 0.4940 (4 d.p.) 49.4% of matches will last shorter than 40 minutes or longer than 50 minutes.	Percentage (or probability) correctly calculated.		
(b)	<ul> <li>Expected coverage:</li> <li>The normal distribution would be an appropriate model because:</li> <li>The shape of the distribution appears to be symmetrical.</li> <li>The shape of the distribution appears to be bell-shaped.</li> <li>The variable is continuous (length of matches measured in minutes).</li> <li>It is possible for lengths of times of the matches to be higher or lower than the bounds shown (although it is very unlikely).</li> <li>Using the normal distribution with μ = 42.2 and σ = 6.8, the probability that the length of the match is between 30 and 55 is P(30 &lt; X &lt; 55) = 0.9337</li> <li>Reading off the histogram, P(30 &lt; X &lt; 55) = 0.09 + 0.23 + 0.30 + 0.24 + 0.07 = 0.93, which is very similar to the theoretical probability obtained using the normal distribution.</li> </ul>	A relevant probability is calculated using the normal distribution. OR Two relevant features of the distribution are described.	TWO relevant features of the distribution are described and compared to the features of the normal distribution. OR A relevant probability is calculated using the normal distribution and compared to the given distribution.	Explanation includes comparison of at least 2 features of the given distribution and the normal distribution. AND At least one relevant calculation is performed to allow comparison of probabilities between the given <b>distribution</b> and the normal <b>distribution</b> with conclusion. [Not just a small part of the distribution].
(c)	Distribution of calories burned for player B: Normal distribution, $\mu = 1496$ , $\sigma = 7.81$ P(X > 1500) = 0.3043 For player C: $P(X > 1500) = 0.28$ As $P(X > 1500)$ is smaller for player C, the standard deviation of the distribution of calories burned by player C is smaller than that of player B OR Find the value of the standard deviation for player C using inverse normal calculations and using this to make the comparison. $z = 0.58284$ $\sigma = 6.863$	Probability for player B correctly calculated. 0.3043 OR Std dev for C correctly calculated $\sigma = 6.863$	Explanation with suitable reasoning given that the standard deviation is smaller. OR $\sigma = 6.9$ with suitable reasoning.	

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	Almost complete correct answer	1 of u.	At least 2 of u.	1 of r.	2 of r.	1 of t (with minor omission).	l of t.

Two	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	<ul> <li>Key features of distribution:</li> <li>discrete random variable</li> <li>lowest value 0, highest value 6 (for the data given, but could be higher)</li> <li>shape is skewed</li> <li>mode is 2.</li> <li>Mean number of goals scored per game is approximately 2.6</li> <li>E(G) = 0 × 0.06 + 1 × 0.18 + 2 × 0.25 + 3 × 0.23 + 4 × 0.16 + 5 × 0.1 + 6 × 0.02 = 2.63 = 2.6 (1 d.p.)</li> </ul>	Two features of the distribution are described. (or Poisson in context). OR A reasonable estimate of the mean is obtained (eg using expected value, using inverse Poisson or from graph).		
(a)(ii)	<ul> <li>Poisson distribution with λ = 2.6</li> <li>Applying this distribution because:</li> <li>discrete (number of goals) within a continuous interval of 60 minutes</li> <li>goals are rare</li> <li>two goals cannot occur at the same time</li> <li>each goal is independent of any other</li> <li>no upper limit on the number of goals scored.</li> <li>Using Poisson with λ = 1.3 (for 30 minutes)</li> <li>P(X ≥ 2) = 1 - P(X ≤ 1)</li> <li>= 1 - 0.6268 = 0.3732</li> </ul>	Probability calculated with identification of (probability distribution and) the parameter.	Probability calculated with identification of probability distribution and the parameter. AND justification of applying this distribution linked to the context for at least TWO conditions.	
(b)(i)	Poisson, $\lambda = 0.8$ P(X < 3) = P(X ≤ 2) = 0.9526 P(both games fewer than three goals) = 0.95262 <sup>2</sup> = 0.9074 Assumption that events "fewer than three goals scored in first game" and "fewer than three goals scored in second game" are independent.	Probability correctly calculated for one game.	Probability correctly calculated for both games supported by assumption in context.	

(b)(ii)	From (b)(i) Poisson, $\lambda = 0.8$ P(X < 3) = P(X \le 2) = 0.9526 So P(X \ge 3) = 1 - 0.9526 = 0.0474 Using binomial distribution with $n = 5$ and $p = 0.0474$ . P(X \ge 2) = 1 - P(X \le 1) = 1 - 0.9796 = 0.0204 Assumptions: • Probability of scoring at least than three goals remains fixed across the five games. • The outcome of a game is not influenced by the outcome of any other game.	Probability of "success" in a single game correctly calculated.	Probability of "success" in a single game correctly calculated. AND correct identification of the binomial distribution as an appropriate model to solve the problem (with parameters $n = 5$ and p = 0.0474).	Binomial distribution is applied correctly to find the required probability, with at least ONE assumption correctly stated in context.
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NO	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start/attempt at one part of the question.	Almost complete correct answer.	l of u	At least 2 of u.	1 of r.	2 of r.	1 of t (with minor omission).	1 of t.

Three	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	P(X > 8) = 1 - P(X < 8) $P(X < 8) = 0.5 \times 6 \times 0.06 = 0.18$ P(X > 8) = 1 - 0.18 = 0.82	Probability correctly calculated.		
(ii)	According to the new data, $P(X < 8) = 0.12$ , which is lower than the model given, which gives $P(X < 8) = 0.18$ .	Identification that mode needs to be higher	New value for mode correctly stated.	
(b)	<ul> <li>Binomial distribution, with n = 20, p = 0.04</li> <li>Binomial because:</li> <li>only two outcomes for each shirt – defective or not defective</li> <li>fixed probability of 0.04 of getting a defective shirt</li> <li>fixed number of 20 shirts (trials)</li> <li>independence of the occurrence of defective shirts, ie assume that the machine does not do "batches" of defective shirts.</li> <li>P(X ≥ 2) = 1 - P(X ≤ 1) = 1 - 0.8103 = 0.1897</li> </ul>	Probability calculated with identification of probability distribution.	Probability calculated with identification of probability distribution and the parameters. AND justification of applying this distribution linked to the context (shirts) for at least TWO conditions.	
(c)	Using binomial distribution as the theoretical distribution, with $p = 0.5$ , as the racket is expected to be balanced. Binomial $(n = 20, p = 0.5)$ $P(X \ge 13) = 1 - P(X \le 12)$ = 1 - 0.8684 = 0.1316 This probability is not very small, which indicates that chance alone could explain the variation in results i.e. we would expect variation in the number of times the racket lands up when spun, and these results fit within this expected variation, especially when taking into account that only a small number of trials were conducted. Therefore, there is not sufficiently strong evidence to claim the racket is unbalanced.	Binomial distribution identified with correct value for <i>p</i> AND A relevant Binomial probability correctly calculated.	Relevant Binomial probability correctly calculated. AND A reasonable attempt to interpret this probability in terms of the complaint.	A complete and well- reasoned explanation is given regarding the complaint made, supported by <i>correct</i> interpretation of the probability calculated.

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start/attempt at one part of the question.	Almost complete correct answer.	1 of u.	2 of u.	l of r.	2 of r.	1 of t (with minor omissions).	1 of t.

## Judgement Statement

	Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
Score range	0 – 7	8 – 12	13 – 18	19 – 24