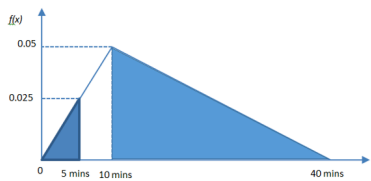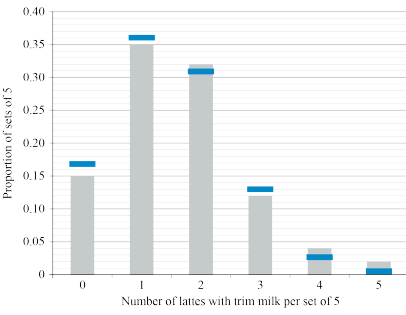**Assessment Schedule – 2014**

Mathematics and Statistics (Statistics): Apply probability distributions in solving problems (91586)

Evidence Statement

One	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	Normal distribution $\mu = 115$, $\sigma = 10$ $P(108 < X < 122) = 0.516$, (0.52 or 0.51 ok) $P(\text{all three coffees}) = 0.516^3 = 0.137$ Assuming that the amounts of **caffeine** in each coffee (latte) are independent.	Correct probability of an individual coffee. OR Incorrect probability of one coffee, consistently cubed and with correct independence statement, in context	Correct probability of all three coffees containing required amount of caffeine, with assumption of independence stated in context.	
(b)	Using a triangular distribution: $P(X > 10) = \frac{1}{2} \times 30 \times 0.05 = 0.75$ (3/4) $P(X < 5) = \frac{1}{2} \times 5 \times 0.025 = 0.0625$ (1/16) $P(X < 5 \text{ or } X > 10) = 0.75 + 0.0625$ $= 0.8125$ (13/16)	Either $P(X < 5)$ OR $P(X > 10)$ correctly calculated.	Required probability is calculated.	
(c)(i)	Normal distribution $\mu = 400$ $P(X > 405) = 0.35$ $z = 0.385$ $0.385 = \dfrac{405 - 400}{\sigma}$ $\sigma = 12.99$ mL $P(X > 410) = 0.221$ or 22% (% not needed)	Calculation of $P(X > 410)$ based on an incorrect or estimated standard deviation (shown). OR Correct z value.	Correct calculation of standard deviation. Must show working used (z value). OR Correct probability of 0.221 using correct standard deviation (no working) plus correct discussion of a limitation in context.	Correct calculation of $P(X > 410)$ with working shown to get standard deviation. AND Discussion clearly identifies one limitation with using the normal distribution, and links this to the context to explain why it is a limitation.
(ii)	The cup for the "large size" latte can be filled up to only a certain volume before the extra amount will spill out. Therefore, there is an upper limit to the amount of milk that can be used in the "large size" latte. A normal distribution does not have an upper limit. Similarly the amount of milk has a lower bound, 0 and the normal distribution has no lower bound. The distribution of milk used may not be symmetrical, since they would be aiming to fill it up to maximum without over filling. The distribution is more likely to be negatively skewed.			

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	Almost complete correct answer	1 of u	2 of u	1 of r	2 of r	1 of t (with minor omission)	1 of t

Two	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	Binomial distribution because: • Only two outcomes for each latte order – with trim milk or not. • Fixed probability of 30% for a latte with trim milk. • Fixed number of trials (sets of five latte orders). • Assume each latte order is independent from another. (Independence could be implied by description).	At least TWO conditions for binomial distribution identified, but are not related to the context.	Justification of applying this distribution linked to the context (lattes with trim milk) for at least TWO conditions.	
(ii)	Binomial distribution $n = 5$, $p = 0.3$ $P(X = 3) = 0.13$ (0.1323) $P(X = 4) = 0.03$ (0.0284) $P(X = 5) = 0.00$ (0.0024) These are plotted on the graph: 	At least one correct probability calculated, graphed or not graphed.	The three distribution probabilities are plotted on the graph. AND ONE feature of the experimental distribution AND the theoretical distribution is described and compared. OR A valid conclusion.	The three distribution probabilities are plotted on the graph. AND ONE feature of the experimental distribution and the theoretical distribution described and compared for E7, AND TWO features of the experimental distribution and the theoretical distribution described and compared for E8. AND A valid conclusion.
(iii)	**Conclusion**:- the binomial model is a good model for this situation. OR The experimental and theoretical distributions are similar. **Discussion:** features of the experimental distribution and the theoretical distribution are described and compared. Eg: • Differences in results, for 0-5, for the two graphs are discussed. • Most common number made or least common number made are identified in both graphs. • Mean of the two distributions compared. • Shapes of both distributions compared, positively skewed. • Correct discussion of sampling variability, (eg another week or another sample would produce different results) *Accept other answers showing understanding of differences/similarities.*			

(b)(i)	Poisson distribution $\lambda = 4.6$ (per hour) $\lambda = 2.3$ (per half hour) $P(X \le 2) = 0.596$	Probability correctly calculated.	Probability correctly calculated. AND At least one invalid assumption identified and discussed in context.	
(ii)	Possible invalid assumptions: • Phone orders occur randomly – more likely that phone orders occur around set times, and not at random throughout the day. • The mean number of phone orders received per hour (or the rate) is constant – the number of phone orders received is more likely to vary with the time of day, eg early morning you would expect a higher rate of phone orders. • Events cannot occur simultaneously – it could be possible for two orders to arrive at the same time, call waiting, multiple phone lines. • Independence – people ordering a coffee, talking about coffee, drinking coffee could influence another person ordering one. *Accept other reasonable specific discussion around why the Poisson model may not be suitable, linked to the conditions required to apply the distribution.*			

NØ	N1	N2	A3	A4	M5	M6	E7	E8
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Three	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	$E(N) = 1 \times 0.49 + 2 \times 0.31 + 3 \times 0.1 + 4 \times 0.08 + 5 \times 0.02$ $= 1.83$	$E(N)$ correctly calculated. OR Consistent calculation of $E(C)$ from an incorrect table.	$E(C)$ correctly calculated.	
(ii)	For orders of 1 or 2 coffees, Cost $= 1.8N + 0.2$ For orders of 3, 4 or 5 coffees, Cost $= 1.8N + 0.4$ <table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>Cost (c)</td><td>2</td><td>3.8</td><td>5.8</td><td>7.6</td><td>9.4</td></tr><tr><td>P(C = c)</td><td>0.49</td><td>0.31</td><td>0.1</td><td>0.08</td><td>0.02</td></tr></table> $E(C) = 2 \times 0.49 + 3.8 \times 0.31 + 5.8 \times 0.1 + 7.6 \times 0.08 + 9.4 \times 0.02$ $= \$3.53$ OR $E(C) = 1.8 \times 1.83 + 0.8 \times 0.2 + 0.2 \times 0.4 = \$3.53$ OR $E(C) = \$3.43$ or $\$3.44$ (based on 1 coffee not needing a tray, so cost is $1.80, instead of $2)			
(b)(i)	Uniform distribution (min = $0.5, max = $5) $P(X < k) = 0.8$ $k = 5 - 0.2 \times (5 - 0.5) = \$4.10$ OR Triangular distribution - min = $0.5, max = $5, a mode of 0.444 or $\frac{4}{9}$, but this mode could be at various places, plus relevant calculation started. OR Normal distribution – mean = 2.75, standard deviation = $\frac{4.5}{6} = 0.75$ or similar, plus relevant calculation started.	1. The donation amount of $4.10 is found using the uniform distribution OR 2. Triangular or normal distribution is used with relevant calculation started. [OR Diagram of model with incorrect calculation or no calculation + correct justification given in context.]	Achievement by 1. or 2. AND Justification for selected model.	
(ii)	Justifications : • A **uniform** distribution can be used because minimum and maximum amounts are given, and there is no mode given. • The **normal** distribution - it would be reasonable to assume that people's generosity would be normally distributed between $0.5 and $5. A few would give very little, a few would give a lot, but most would be around the middle. • The **triangular** distribution could be used since we have minimum and maximum amounts. It is reasonable to assume there will be a mode value.			

(c)	Poisson distribution $P(X \geq 1) = 0.96$ $P(X = 0) = 0.04$ $e^{-\lambda} = 0.04$ $\lambda = 3.22$ (per 15 minutes) $\lambda = 6.44$ (per 30 minutes) $P(X > 10) = 1 - P(X \leq 10) = 1 - 0.9365 = 0.0635$ *Accept the use of tables to find $\lambda = 3.2$.*	Consistent calculation of $P(X > 10) = 1 - P(X \leq 10)$ using an incorrect λ	Calculation of $\lambda=3.2$	Calculation of the probability of more than 10 visits to the website. **NB** minor error with $\lambda = 6.44$ is $1 - P(X < 10)$ $= 1 - 0.8824$ $= 0.1176$

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Cut Scores

	Not Achieved	**Achievement**	**Achievement with Merit**	**Achievement with Excellence**
Score range	0 – 8	9 – 13	14 – 19	20 – 24