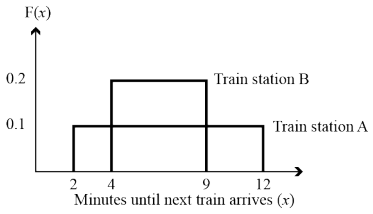
Assessment Schedule – 2015

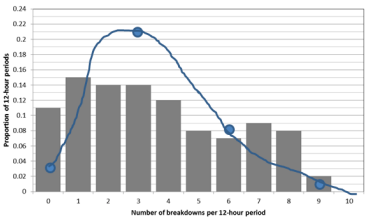
Mathematics and Statistics (Statistics): Apply probability distributions in solving problems (91586)

Evidence Statement

One	Expected Coverage	Achievement, (u)	Merit, (r)	Excellence, (t)
(a)(i)		One rectangle drawn correctly, including correct height. OR Two rectangles drawn, without heights identified, but rectangle B is twice the height of rectangle A.	Both rectangles drawn correctly, including correct heights.	
(ii)	For train station A $P(X > 8) = 0.4$ For train station B $P(X > 8) = 0.2$ $P(\text{both trains}) = 0.4 \times 0.2 = 0.08$ Assuming the arrival times for the two trains are independent events.	Correct probabilities calculated for different train stations.	Correct joint probability calculated. AND Assumption given in context.	
(b)(i)	Binomial, n = 7, p = 0.13 $P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.772$ $= 0.228$	Correct probability calculated for (i).	Correct probability calculated for (i). AND Model identified as binomial and justified with at least two conditions linked to the context.	
(ii)	Binomial because: • fixed number of trials (7 cars) • fixed probability of success (13% red) • only two outcomes (red or not red) • independent events (colour of one car should not affect colour of another car).			
(iii)	$P(X \geq 1) = 0.965$ $P(X = 0) = 0.035$ $0.87^n = 0.035$ $n = 24.07$ So 24 cars (accept 25 cars) *Accept use of trial and improvement with graphics calculator if supporting explanation is given.*			Correct number of cars determined for (iii) with clear communication of strategy used to solve problem.

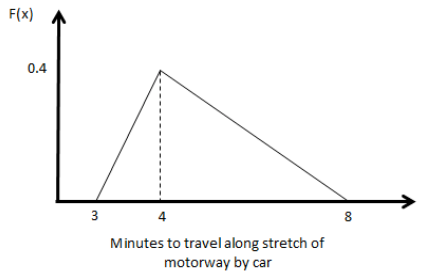
NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start/attempt at one part of the question.	Almost complete correct answer.	1 of u	2 of u	1 of r	2 of r	1 of t (with minor omission or error).	1 of t

Two	Expected Coverage	Achievement, (u)	Merit, (r)	Excellence, (t)
(a)(i)	E(N) $= 1 \times 0.82 + 2 \times 0.14 + 3 \times 0.03 + 4 \times 0.01$ $= 1.23$ ***Note:*** *Do not accept whole number answers.*	Expected value correctly calculated.		
(a)(ii)	Expected cost for test based on 1.23 attempts = $1.23 \times \$137 = \$168.51$ Total expected cost = \$468.51 Difference = \$300 So \$300 is the fixed cost charged.		Fixed price correctly calculated.	
(b)(i)	Poisson, $\lambda = 1.2$ (breakdowns per four hours) $P(X \leq 2) = 0.879$ Assuming: • constant rate of breakdowns for any four-hour period • breakdowns occur randomly • breakdowns occur independently. • Breakdowns cannot occur simultaneously	Probability correctly calculated.	Probability correctly calculated. AND Two assumptions given in context.	

(ii)	Discussion of assumptions of Poisson model: Not reasonable to assume the rate of breakdowns is constant for any 12-hour period – likely to be differences between day and night "shifts", due to how many buses are on the roads during these times. Comparison of features of experimental and model (theoretical) distributions: Centre: Model (theoretical) distribution has centre (mean) of 3.6 breakdowns – experimental distribution appears to have its centre around 3.6 (visual estimate). Spread: Model (theoretical) distribution mean is equal to variance – experimental distribution appears to have a greater standard deviation than root(3.6) = 1.9 breakdowns. Shape: Model (theoretical) distribution is unimodal – experimental distribution appears bimodal. Shape: Visual comparison by drawing model (theoretical) distribution over experimental distribution graph (see below).  Overall, it appears the Poisson model is not a good model for the distribution of bus breakdowns for any 12-hour period.	One comparison is made between the experimental and model (theoretical) distributions. (Numerical values (e.g. probabilities, calculations or estimates of mean or standard deviation) need not be used)	One comparison is made between the experimental and model (theoretical) distributions with some numerical values used (e.g. probabilities, calculations or estimates of mean or standard deviation). AND An overall conclusion as to the goodness of the model.	ONE discussion point based on comparisons between the experimental and model (theoretical) distributions with numerical values used (e.g. probabilities, calculations or estimates of mean or standard deviation). AND At least one discussion point which could be either An assumption of the Poisson model, discussed in context OR Another comparison between the experimental and model (theoretical) distributions as above AND An overall conclusion as to the goodness of the model.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start/attempt at one part of the question.	Almost complete correct answer.	1 of u	2 of u	1 of r	2 of r	1 of t (with minor omission or error).	1 of t

Three	Expected Coverage	Achievement, (u)	Merit, (r)	Excellence, (t)
(a)(i)	Normal, $\mu = 7500$, $\sigma = 2000$ $P(X > 9000) = 0.227$ $P(X > 9500) = 0.159$ $P(X > 9500 \mid X > 9000)$ $= 0.159 \div 0.227$ $= 0.700$ 70% of the "over-priced" cars were expected to have been sold for more than \$9500.	Probability of either single event correctly calculated.	Conditional probability correctly calculated.	
(a)(ii)	Possible discussion points: Shape of sample distribution is only slightly negatively skewed with the potential outliers – two of the cars were advertised at much lower prices than the other 47 cars, causing the impression of negative skew (longer LHS tail). Sample is based on advertised selling prices, not the actual selling prices. This might account for the skewness (some cars likely to sell for lower). Sample is from one trading website – which may not be representative of NZ prices Size of sample – can't tell shape of underlying distribution with such small sample of cars. *Accept discussions based on the skewness or shape of the sample distribution in comparison to the model (theoretical) distribution.*	One possible reason why the shape of the sample distribution could be different from the shape of the underlying population distribution is clearly identified but not fully discussed.	One possible reason why the shape of the sample distribution could be different from the shape of the underlying population distribution is fully discussed.	Two possible reasons why the shape of the sample distribution could be different from the shape of the underlying population distribution are fully discussed.

(b)(i)	Triangular distribution.  $P(X > 4) = 0.5 \times 4 \times 0.4 = 0.8$	Probability correctly calculated in (i) or (ii).	Suitable evidence (e.g. a relevant probability calculation) is used to explain why 6 minutes is not the median.	
(b)(ii)	For 6 minutes to be the median, 50% of times travelled would need to be greater than 6 minutes. At $x = 6, F(x) = \frac{1}{2} \times 0.4 = 0.2$ (using similar triangles) $P(X > 6) = 0.5 \times 2 \times 0.2 = 0.2$ As this is not 50%, 6 minutes cannot be the median. At $x = 6,\ F(x) = \frac{1}{2} \times 0.4 = 0.2$ (using similar triangles) $P(X > 6) = 0.5 \times 2 \times 0.2 = 0.2$			

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Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 8	9 – 13	14 – 18	19 – 24