Assessment Schedule – 2019

Mathematics and Statistics (Statistics): Apply probability distributions in solving problems (91586)

Evidence Statement

<table>
<tr><th>Q</th><th>Expected coverage</th><th>Achievement (u)</th><th>Merit (r)</th><th>Excellence (t)</th></tr>
<tr><td>ONE (a)(i)</td><td>Binomial distribution, $n = 10$, $p = 0.45$
$P(X \geq 6)$
$= 1 - P(X \leq 5)$
$= 1 - 0.7384$
$= 0.2616$</td><td rowspan="2">Probability correctly calculated.

OR

Assumption identified and discussed why it may be invalid in context.</td><td rowspan="2">Probability correctly calculated for (a)(i).
AND
In context, assumption identified and discussed why it may be invalid in context.</td><td rowspan="2">Probability correctly calculated for (a)(i).
AND
In context, assumption identified and discussed why it may be invalid.
AND
Impact of invalid assumption described.</td></tr>
<tr><td>(ii)</td><td>Binomial distribution may be invalid:
• One zucchini seed growing into a plant will not impact another zucchini seed growing into a plant. This may be invalid as conditions like weather, birds, frequency of watering will impact all the seeds in the garden bed. (independence).
• The probability that each zucchini seed will grow into a plant is the same (0.45). This may be invalid as there are sometimes big healthy looking seeds in a packet and some tiny seeds so presumably the rate of growing into a plant may be different. (Probability of success remains constant).
• The probability is given for the previous year, but conditions this year may be different, so the probability of growing into a plant may be different this year.
Discussion of impact of making this assumption, e.g. many more (or less) seeds may grow into a plant than calculated, hence probability calculated may be an overestimate or an underestimate.
Accept other valid discussions.
Do not accept discussion around fixed number of trials or two outcomes being invalid.</td></tr>
<tr><td>(b)(i)</td><td>Triangular distribution, $a = 190$, $b = 270$, $c = 220$
Height at mode = $\dfrac{2}{270 - 190} = 0.025$ or $\dfrac{1}{40}$
$P(X < 220) = 0.5 \times (220 - 190) \times 0.025 = 0.375$</td><td>Probability correctly calculated.</td><td></td><td></td></tr>
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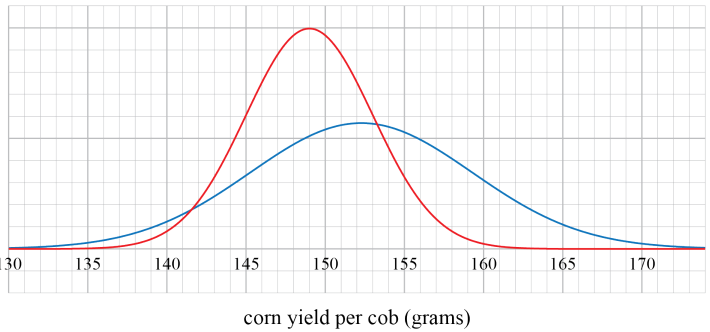
(ii)	This probability is less than 0.5; therefore 220 is not the median. The median is greater than 220 as the P(X < median) must be 0.5. *Mark for consistency with (b)(i).*	Reasoned statement with evidence that the median must be greater than 220.	Reasoned statement with evidence that the median must be greater than 220. AND Comparison of calculated probability with 0.5.	
(iii)	Triangular distribution, $a = 190$, $b = 270$, $c = 220$ $P(X < 215) = \frac{1}{2} \times \left(\frac{25}{30} \times \frac{1}{40}\right) \times 25 = 0.2604$ accept rounding to 0.26. Binomial, $n = 10$, $p = 0.26$ $P(X \leq 5) = 0.9759$ Assumptions: • Each plant produces zucchinis of similar size (the probability of a zucchini being less than 215 mm remains constant). • The length of one zucchini does not influence the length of any other zucchini. Note that those candidates without graphics calculators will use $p = 0.25$, giving $P(X \leq 5) = 0.9803$.	Probability for one zucchini correctly calculated ($p = 0.26$). OR Calculated binomial using incorrect probability.	Probability for one zucchini correctly calculated. AND Correct identification of the binomial distribution. OR Binomial distribution used to calculate required probability.	Probability for one zucchini correctly calculated. AND Binomial distribution used to calculate required probability with parameters stated. AND At least ONE assumption correctly stated in context.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Q	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)(i)	$\frac{9}{62}$ is 14.5%, which is more than 10%; therefore claim is correct. OR 10% of 62 = 6.2 9 cobs had more than 160 grams; therefore claim is correct. Do not accept use of the normal distribution.	Proportion correctly calculated AND a statement that claim is correct.		
(ii)	Expected coverage: The Normal Distribution model would be an appropriate model for the yield per cob because: • The shape of the distribution appears to be symmetrical about the mean (152.3g), unimodal and somewhat bell-shaped. This makes sense as most yields would be close to the mean with roughly equal proportions above and below the mean • Probability calculation(s) given comparing theory vs observed e.g. Model P(X>160g)=0.1357 which is close to Observed P(X>160g)=0.1452 • Discussion or calculation of standard deviation or range or middle 50% or similar • Slight differences observed due to sampling variation. As 62 is a relatively small sample, we would not expect the observed distribution to perfectly match the theory. If we had another sample of 62, it would look slightly different.	ONE of a relevant feature of the distribution is described and compared to the features of the normal distribution. OR A relevant probability is calculated using the normal distribution and compared to the given distribution.	Either TWO different relevant features of the distribution are described and compared to the features of the normal distribution. OR ONE feature of the distribution are described and compared to the features of the normal distribution and a relevant probability is calculated using the normal distribution and compared to the given distribution.	TWO different relevant features of the distribution are described in context and compared to the features of the normal distribution. AND A relevant probability is calculated using the normal distribution and compared to the given distribution.
(iii)	 Height of curve is higher than that for Super-Sweet corn.	Bell-shaped curve with correct centre.	Bell-shaped curve with correct centre, spread and height higher than for 'Super-Sweet'.	

(iv)	Mean yield per cob for Ambrosia-Sweet corn is 149 g which is 3.3 g lower than the mean yield per cob for Super-Sweet corn.(152.3 g). This means that Ambrosia-Sweet corn would typically produce slightly less corn per cob than the Super Sweet corn would. The standard deviation of the corn yield per cob of Ambrosia-Sweet corn (4 g) is also lower than the standard deviation for Super-Sweet corn (7 g). This suggests that Ambrosia-Sweet corn's yield per cob is slightly more consistent than the Super-Sweet corn's yield per cob. Super-Sweet corn might have some very large cobs but also some very small cobs. *Accept other valid comparisons*	ONE valid comparison of the two distributions discussed in context with evidence.	TWO valid comparisons of the two distributions discussed in context with evidence OR ONE valid comparison of the two distributions discussed in context with evidence AND Related back to what this means in context.	TWO valid comparisons of the two distributions discussed in context with evidence AND Related back to what this means in context.
(b)	25% of 'Bush Road' potatoes are less than 186 grams. Potatoes that are 186 grams are 235 – 186 = 49 grams lighter than the mean. Using symmetry, 25% of potatoes will be more than a weight that is 49 grams heavier than the mean (235 + 49 = 284 grams). As 25% of potatoes will be more than 284 grams, less than 25% of potatoes will weigh more than 300 grams. Accept use of the normal distribution: $\mu = 235$ $P(X < 186) = 0.25$ $z = -0.6745$ $-0.6745 = \frac{186-235}{\sigma}$ $\sigma = 72.65$ grams $P(X > 300) = P(Z > 0.894) = 0.186 = 18.6\%$ Less than 25% of potatoes will weigh more than 300 grams.	Calculation (or diagram) showing that 300 g potatoes are more than 49 g heavier than the mean. OR Finds $\sigma = 72.65$ using Normal.	Calculation (or diagram) showing that 300 g potatoes are more than 49 g heavier than the mean. AND Reasoning that clearly identifies why the % is not around 25%.	

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No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Q	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)(i)	Poisson distribution $\lambda = 6$ weeds per square metre $P(X > 8)$ $= 1 - P(X \leq 8)$ $= 1 - 0.8472$ $= 0.1528$	Probability correctly calculated.		
(ii)	Poisson distribution $\lambda = 6$ weeds per square metre $P(X \geq 4)$ $= 1 - P(X \leq 3)$ $= 1 - 0.1512$ $= 0.8488$ $P(X \geq 4)$ over 5 separate square metres $= 0.8488^5$ $= 0.4406$ OR Use of binomial distribution $n = 5$, $p = 0.8488$ $P(X = 5) = 0.4406$	Probability of $P(X \geq 4)$ correctly calculated. OR Incorrect probability used correctly to find probability.	$P(X \geq 4)$ over five square metres correctly calculated. AND Correct assumption identified in context.	Probability from (ii) correct.
(iii)	Assumption made that the number of weeds in each of the 5 separate square metres is independent. Discussion in context of whether (or not) this assumption is valid E.g. if the 5 separate square metres are close to each other, then the weeds may spread between the plots, meaning that the number of weeds in each plot may not be independent. If binomial distribution is used in (ii), accept assumption of constant probability of each square metre having at least 4 weeds OR independence of each square metre with at least 4 weeds.	AND Assumption in context.	OR Correct assumption identified in context. AND Correct discussion of the validity of an assumption in context.	AND Correct assumption identified in context. AND Correct discussion of the validity of an assumption in context.

(b)(i)	Using data: Total number of weeds $= (1 \times 1) + (5 \times 2) + (6 \times 3) + (7 \times 4) + (5 \times 5) + (3 \times 6) + (1 \times 7) + (1 \times 8) + (1 \times 10) = 125$ Mean number of weeds $= \frac{125}{30} = 4.1667$ Calculations not required. *Do not accept use of the Poisson distribution.*	Mean = 4.2 Correct answer only.		
(ii)	$P(X = 10)$ using $\lambda = 4.1667 \times 3 = 12.5$ (1dp) $= 0.0956$ Poisson because: • Discrete event in a continuous interval of space: number of weeds per 1 m^2. • Random – weed growth occurs randomly and unpredictably. • Independent – one weed growing will not influence the growth of another. • Probability of a weed growing is proportional to the area (1m^2) OR the number of weeds per three square metres will be found at a constant rate • Events cannot occur simultaneously – 2 weeds can't grow in the same place. *Solution consistent with (b)(i) where a mean that is realistic for the distribution is used.*	Proportion calculated correctly for: $\lambda = 4.2$ OR $\lambda = 12.5$	Proportion correctly calculated for $\lambda = 12.5$ AND Choice of distribution justified.	
(iii)	The original rate was 6 weeds per square metre, now it is 4.2 weeds per square metre. 40% reduction of 6 is 3.6 weeds per square metre, (and after using the weed killer, there are 4.2 weeds per square metre) so there is not a reduction rate of at least 40%. OR $\frac{1.8333}{6} = 30.5\%$ reduction, so there is not a reduction rate of at least 40%.		Correct calculation of percentage reduction.	Correct comparison with original rate, concluding that the weed reduction rate for the new weed killer is not at least 40%.

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Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 14	15 – 19	20 – 24