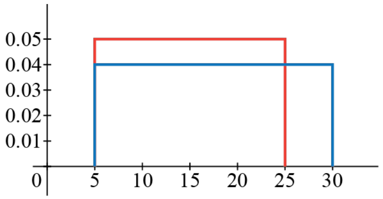
Assessment Schedule – 2020

Mathematics and Statistics (Statistics): Apply probability distributions in solving problems (91586)

Evidence Statement

Q	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)(i)	 Red → lunch Blue → dinner	Both rectangles drawn correctly, including correct heights and clearly identified.		
(ii)	P(cold lunch AND cold dinner) = P(cold lunch) × P(cold dinner) $= (5 \times 0.05) \times (10 \times 0.04)$ $= 0.1$	Probability correct for one cold meal.	Probability correctly calculated for lunch AND dinner being cold.	
(iii)	Assuming that the event "a patient's lunch is cold" is independent of the event "a patient's dinner is cold". This assumption may not be valid, as a patient is unlikely to have moved rooms from lunch to dinner and if their room is a long way from the kitchen then they are more likely to have cold meals both times. *Accept other reasonable statistical discussions for assumption being valid or not.*		Correct assumption(s) identified in context. OR Correct discussion of the validity of assumption(s) in context.	Correct assumption(s) identified in context. AND Correct discussion of the validity of assumption(s) in context.
(b)(i)	Poisson distribution $\lambda = 3.2$ vegetable servings per day $P(X \geq 3) = 1 - P(X \leq 2)$ $= 1 - 0.3799$ $= 0.6201$	Probability correctly calculated $P(X \geq 3)$.		

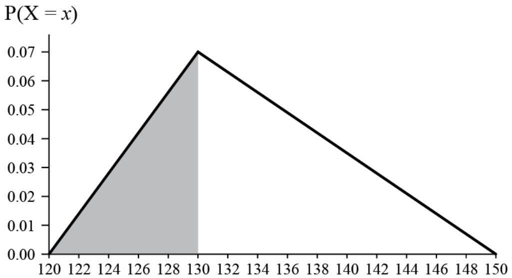
(ii)	P(X = 0) = 0.05 and calculating λ: $e^{-\lambda} = 0.05$ $\lambda = -\ln 0.05$ $\lambda = 2.996$ So $\lambda = 3.0$ (1dp)	Setting up relevant equation.	Showing clearly how λ = 3.0 is found.	
(iii)	Assuming that each occurrence of eating a serving of vegetables is independent of other occurrences of eating a serving of vegetables may be invalid, because vegetables are often eaten as part of a full meal, and often served together. For example, if the meal was a roast then it is common to have more than one roast vegetable: potatoes, pumpkin, kumara, carrots, etc. Assuming that events "eating a serving of vegetables" cannot happen simultaneously may be invalid because, if there is more than one serving of vegetables in a meal, people might mix the vegetables together to take a mouthful, essentially eating the servings of vegetables at the same time. For example, if a meal has both potatoes and peas, a person might eat a mouthful of potatoes WITH their peas. Assuming that events "eating a serving of vegetables" occur at random and are unpredictable may be invalid because it is common for vegetables to be served with dinner, not consumed unpredictably throughout the day. Assuming that events "eating a serving of vegetables" occur at a constant rate throughout the day may be invalid because you consume more vegetables at lunch than you do at breakfast (you don't usually eat vegetables at breakfast) *Accept other correct discussions.*	One correct assumption identified in context.	TWO correct assumptions identified in context. OR ONE correct assumption identified in context with reasoning as to why it is not appropriate.	TWO correct assumptions identified in context with reasoning as to why it is not appropriate.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Q	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)(i)	Binomial $n = 8$, $\pi = 0.499$ (accept use of $\pi = 0.5$ for tables) (using calculator) $P(X < 4) = 0.3655$ Or (using tables) $P(X < 4) = 0.0039 + 0.0313 + 0.1094 + 0.2188$ $= 0.3634$	Probability correctly calculated.		
(ii)	Binomial $n = 10$, $\pi = 0.499$ (accept use of $\pi = 0.5$ for tables) $P(X = 5)$ $= 0.2461$ Student is incorrect, because the chance of exactly 5 of the next 10 students sampled eating breakfast daily is only 24.61%, which is quite low.	Probability correctly calculated.	Probability correctly calculated. AND Compared to the claim with conclusion.	
(iii)	The variation in the number of individuals who ate breakfast daily can be measured with the standard deviation. The standard deviation of the number of children eating breakfast daily is $\sqrt{10 \times 0.85 \times 0.15} = 1.13$ The standard deviation of the number of youths eating breakfast daily is $\sqrt{10 \times 0.499 \times 0.501} = 1.58$ *(Accept use of 0.5 rather than 0.499).* The standard deviation for the number who ate breakfast daily is greater for the youths than the children – the health worker's observation can be justified statistically. OR The VAR of the number of children eating breakfast daily is VAR= 1.2769 The VAR of the number of youths eating breakfast daily is VAR= 2.4964 The variation for the number who ate breakfast daily is greater for the youths than the children – the health worker's observation can be justified statistically.	Calculation of at least one correct standard deviation or variance.	Calculation of both correct standard deviations or both variances.	Calculation of both correct standard deviations OR both variances. AND Conclusion that as the standard deviation or variance of the number of youths is greater than that for the children (for samples of size 10), then the health worker's observation is justified.

(b)(i)	Normal distribution, $\mu = 11\,200$ kJ, $\sigma = 2230$ kJ $P(X > 12\,800) = 0.23653627$ (Using tables = 0.2367)	$P(X > 12\,800)$ correctly calculated using normal distribution model.		
(ii)	Using the normal distribution model with parameters, $\mu = 11\,200$ kJ and $\sigma = 2230$ kJ $P(9500 < x < 12\,000) = 0.4172$ The given model is not valid, since the observed probability (0.5) is much higher (20%) than the model predicts. Hence, the normal distribution model presented above is not appropriate for modelling the energy intake at breakfast of New Zealand male youths.	Calculation of $P(9500 < x < 12\,000)$	Reasoning that the model under-predicts the observed probability (0.5), therefore it is not appropriate.	
(iii)	Proposed mean, $\mu = 11200$ kJ The same as that for all New Zealand youths (no evidence to suggest any other value). Potential calculation. $z^{-1}{}_{0.35} = 1.036$ $\sigma = \dfrac{12\,800 - 11\,200}{1.036} = 1544.4$ Proposed standard deviation, $\sigma = 1544$ kJ Proposed standard deviation is less than that of the current model (2230 kJ) because the probability of a youth at this boys-only school eating over 12800kJ (15%) is less than that for all New Zealand youths (24%).		Suitable values for one parameter are given and used to calculate the second	EITHER Suitable values for one parameter are given and used to calculate the second AND Assumption for the fixed parameter stated and then linked to changed parameter OR Comparing probability for NZ male youths with single sex school (24%>15%) and linking the probability to original parameter and justify new model and its parameters.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Q	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)(i)	 Triangular distribution with a = 120, b = 150, c = 130 Height at X = 130: $h=\frac{2}{150-120}=\frac{1}{15}=0.0666667$ $\text{P}(\text{X}<130)=\frac{1}{2}\times 10\times\frac{1}{15}=\frac{1}{3}=0.333333333$	Probability correctly calculated P(X < 130).		
(ii)	Height at X = 140: $h=\frac{2(150-140)}{(150-120)(150-130)}=\frac{1}{30}$ $\text{P}(\text{X}>140)=\frac{1}{2}\times 10\times\frac{1}{30}=\frac{1}{6}$ $\text{P}(\text{X}<140)=1-\frac{1}{6}=\frac{5}{6}$ $\text{P}(\text{X}>130 \mid \text{X}<140)=\frac{\text{P}(130<\text{X}<140)}{P(\text{X}<140)}$ $=\frac{0.5}{0.83333}$ $=0.6$ OR $\text{P}(\text{X}>130 \mid \text{X}<140)=\frac{\frac{5}{6}-\frac{1}{3}}{\frac{5}{6}}=\frac{3}{5}=0.6$	$\text{P}(\text{X}\geq 140)$ correctly calculated. OR P (X < 140) correctly calculated.	Probability of surgery lasting between 130 and 140 minutes correctly calculated.	Conditional probability correctly calculated.

(b)(i)	<table><tr><td>Number of repairs (N)</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Probability</td><td>0.11</td><td>0.34</td><td>0.35</td><td>0.2</td></tr></table> Let the random variable number of repairs be N. Mean, E(N) $= 0 \times 0.11 + 1 \times 0.34 + 2 \times 0.35 + 3 \times 0.2$ $= 1.64$ $\text{VAR}(N) = \text{E}(N^2) - [\text{E}(N)]^2$ $\text{E}(N^2) = 3.54$ $\text{VAR}(N) = 3.54 - 1.64^2$ $\text{VAR}(N) = 0.8504$ $\text{SD}(N) = 0.9222$	E(N) AND SD(N) correctly calculated.		
(ii)	Company B has higher variation in total lease costs over the three-year period. Additionally, Company A has no variation in total lease costs over the three-year period, as the lease price is fixed.		Correctly states Company B. AND Reasoning that Company A has fixed costs with no variation.	
(iii)	Total cost of leasing for three years from Company A is $C_A = 69500 + 36 \times 350 = \$82\ 100$ Total cost of leasing for three years from Company B is $E(C_B) = 65000 + 1.64 \times 10000 = \$81\ 400$ As the cost for leasing the x-ray unit from Company B is less than the expected cost for leasing the x-ray unit from Company A, the hospital should lease from Company B but Company B's cost could be greater depending on the number of repairs (between \$65000 up to \$95000) so they should choose Company A as its is less risky. (choosing Company B is a risky option as the number of repairs per three years could be 3 which would result in leasing from Company B, costing \$95 000 over the three years.) *Accept valid arguments for Company B.*	Correctly calculates C_A. OR $E(C_B)$ correctly calculated.	Correctly calculates C_A. AND Correctly calculates $E(C_B)$. AND Draws consistent conclusion justified by discussion of cost OR variation in cost.	Correctly calculates C_A. AND correctly calculates $E(C_B)$. AND Draws consistent conclusion justified by discussion of cost AND variation in cost.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	8 – 13	14 – 18	19 – 24