Assessment Schedule – 2023

Mathematics and Statistics (Statistics): Apply probability distributions in solving problems (91586)

Evidence Statement

Q	Expected coverage	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
ONE (a)(i)	Poisson distribution, $\lambda = 9$ $P(X > 10) = 1 - P(X \leq 10)$ $= 1 - 0.70598832$ $= 0.294011$	• Probability correct AND distribution and parameters stated.		
(ii)	Some reasons may be • The location of sheep per hectare may not be random and unpredictable, as their flocking behaviour will mean that you can predict where sheep are likely to be, so the Poisson distribution is not appropriate. • The location of one sheep in a particular hectare may affect the location of other sheep (independence as sheep flock together, where one sheep is, others are also likely to be nearby, so the Poisson distribution is not appropriate. • The average rate at which sheep are found may not be constant / proportional over a different area of land. Due to their flocking behaviour some hectares are likely to have more sheep in them and others are likely to have no sheep, so the rate of sheep per hectare will not be constant but will vary from hectare to hectare, so the Poisson distribution is not appropriate. • NB simultaneous argument supports Poisson.	• ONE correct reason identified in context.	• TWO correct reasons identified in context. OR ONE correct reason identified in context with reasoning as to why use of Poisson is not appropriate.	• TWO correct reasons identified in context with reasoning as to why use of Poisson is not appropriate.
(b)(i)	$P(X > 35) = 25 \times \frac{1}{40} = \frac{5}{8} = 0.625$ $a = 20$, $b = 60$	• Probability correct and parameters stated.		

(ii)	The simulation shows evidence that the proposed uniform model is appropriate to model wool fibre fineness for Awassi sheep. The uniform model assumes that each outcome is equally likely. Each wool fibre class in the observed data appears in the grey cloud of variation that the simulation model produces confirming that the uniform model is appropriate.	• Comment that the uniform model is appropriate or not appropriate supported by reference to observed frequencies and features of the unform model.	• Comment that the proposed uniform model is appropriate supported by references to the variation observed in the results of the simulation model and linked to the original real data	• Comment that the proposed uniform model is appropriate supported by references to the variation observed in the results of the simulation model and linked to the original real data AND linking this variation to the equally likely outcomes of the uniform model.
(iii)	For one sheep: Uniform $a = 20$, $b = 60$ $P(25 < X < 40) = \frac{15}{40} = \frac{3}{8} = 0.375$ For four sheep: $0.375^4 = 0.0198$	• Probability $P(25 < X < 40)$ correct for one sheep. OR Incorrect probability $(\ )^4$ correctly.	• Probability correct for four sheep.	
(iv)	The event in question is "wool fineness is between 25 and 40 microns for one sheep". This independence of event assumption may not be valid, if, for example the sheep are closely related to each other, then it is more likely to have a wool fineness of between 25 and 40 microns or more for multiple sheep. *Accept other valid arguments including it may be independent if sheep are randomly selected from different areas and so not connected genetically.* *Note: other factors such as nutrition, and climate can affect the fineness of wool.*	• Correct event identified in context. OR Correct discussion of the validity of the independence of the event "wool fineness" in context	• Correct event identified in context. AND Correct discussion of the validity of the independence of event assumption.	

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Q	Expected coverage	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
TWO (a)(i)	$P(X = 0) = 0.0027$ and calculating λ: $e^{-\lambda} = 0.0027$ $\lambda = -\ln 0.0027$ $\lambda = 5.9145$ $\lambda = 5.91$	• Setting up relevant equation $e^{-\lambda} = 0.0027$	• Correct calculation for lambda (λ).	
(ii)	$X = 5$ is the most likely outcome because with $\lambda = 5.91$ $P(X = 4) = 0.1379$ $P(X = 5) = 0.16296$ $P(X = 6) = 0.1605$	• Correct value of most likely outcome.	• Correct value of most likely outcome justified by a comparison of probabilities.	
(iii)	Binomial distribution, $n = 10, p = 0.65$ $P(X \geq 9) = 1 - P(X \leq 8)$ $= 1 - 0.91405$ $= 0.08598$ $= 0.0860$	• Probability correctly calculated.		
(b)(i)	Triangular distribution $a = 0.1$ (accept values between 0.05 and 0.1 for *a)* $b = 0.35$ $c = 0.16$ The amount of fertiliser applied is likely to have fixed endpoints – the triangular distribution gives zero probability of applying less than 0.1 (or 0) tonne per hectare or more than 0.35 tonne per hectare. Fertiliser application use is continuous.	• Correct distribution chosen with parameters. OR Reason given for choice of distribution which links to observed distribution in context.	• Correct distribution with parameters AND reason given for choice of distribution which links to observed distribution in context.	• Correct distribution with parameters AND reason given for choice of distribution which links to observed distribution in context. AND One further justification in context.

(ii)	Height at $X = 0.16$ $h = \frac{2}{0.35 - 0.1} = \frac{2}{0.25} = 8$ $P(X < 0.16)$ $= 0.5 \times (0.16 - 0.1) \times 8$ $= 0.24$ Binomial $n = 10$, $p = 0.24$ $P(X > 5) = 1 - P(X \leq 5) = 1 - 0.9838837$ $= 0.0161$ *Or consistent with the value of a between 0.05 and 0.1 from (i).*	• Probability correctly calculated for $P(X < 0.16)$. OR Using incorrect probability to calculate Binomial probability.	• Probability for $P(X < 0.16)$ AND an attempt to calculate final probability.	• Probability correctly calculated for more than half of 10 farms.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Q	Expected coverage	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
THREE (a)(i)	Using the given parameters $P(X < 4290) = 0.4763728 = 0.4764$ This is much larger than 30%. Therefore, the parameters given are not suitable for modelling the milk production of South Island dairy cows. *Accept equivalent methods.*	• Correct probability and comparison with conclusion.		
(ii)	Normal distribution $\mu = 5250$ kg Accept any value between 5000 and 5500. Use inverse normal to calculate σ using $P(X < 3600) = 0.1$ and proposed μ: $\sigma = 1288$ kg Alternatively, using rule of thumb: $\sigma = (9000–2000) / 6$ = 1167 kg *Accept between 1167 and 1390 kg with justification.*	• Correct mean or standard deviation given. OR Using an incorrect parameter to correctly calculate the other.	• Both parameters correctly identified with supporting evidence.	
(iii)	NZ: $\mu = 4370$ $\sigma = 1350$ Herd: $\mu = 5250$ $\sigma = 1288$ Mean milk production per season per cow for NZ dairy cows is 4370 kg, which is lower than the mean milk production per season per cow for the herd of 200 cows (5250 kg) who would typically produce slightly more milk per season per cow than NZ dairy cows. The standard deviation of the milk production per season per cow for NZ dairy cows (1350kg) is slightly higher than / similar to the standard deviation for the herd of 200 cows (1288 kg). This suggests that the consistency of milk production per season per cow is similar for the two groups of cows. *Accept answers consistent with valid means and standard deviations given in part (ii).* *Accept other valid comparisons.*	• ONE valid comparison of the two distributions discussed in context with evidence. OR TWO valid comparisons of the distributions with only context or only numerical evidence.	• TWO valid comparisons of the two distributions discussed in context with evidence. OR ONE valid comparison of the two distributions discussed in context with evidence AND related back to what this means in context.	• TWO valid comparisons of the two distributions discussed in context with evidence AND related back to what this means in context.

(b)(i)	Mean = 1.72 Standard deviation = 0.7082	• Mean and standard deviation correct.		
(ii)	The mean number of kids is less than two, therefore this breed of goats might not be suitable for dairy goat farmers. The standard deviation is 0.71. Farmers need to be aware that there is variability in the number of kids born. It is not unusual for one kid to be born rather than two.		• Comparison with two and conclusion in context.	
(iii)	Change in mean: • There is a higher concentration of proportions for two or more kids per pregnancy, therefore the expected value will increase. Change in standard deviation: The probability of the most common number of goat kids is now reduced to 0.53 from 0.75. The proportions of the number of kids being born used to be mostly concentrated around 2 but now they are more evenly spread across 1 to 4 goat kids, meaning the standard deviation will increase. *Note: No credit awarded for calculations. Explanations must come from description of distributions.*	• Correct explanation for expected value.	• Correct explanation for standard deviation.	• Correct explanation for BOTH expected value and standard deviation.

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Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 13	14 – 18	19 – 24