91586

915860

NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

3

SUPERVISOR'S USE ONLY

Tick this box if there is no writing in this booklet

Level 3 Mathematics and Statistics (Statistics) 2020

91586 Apply probability distributions in solving problems

9.30 a.m. Wednesday 18 November 2020
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

ASSESSOR'S USE ONLY

QUESTION ONE

ASSESSOR'S USE ONLY

(a) A hospital aims to deliver lunch (the midday meal) to patients anywhere between 5 and 25 minutes after the meal has left the kitchen. They also aim to deliver dinner (the evening meal) to patients anywhere between 5 and 30 minutes after the meal has left the kitchen.

(i) Sketch the two probability distribution models for lunch and dinner on this grid.

Clearly identify which is lunch and which is dinner.

If you need to redraw your models, use the grid on page 11.

(ii) Meals are cold if they arrive more than 20 minutes after leaving the kitchen.

Using appropriate probability distribution models, calculate an estimate for the probability that a patient has both a cold lunch and a cold dinner on the same day.

(iii) Consider an assumption you made when you calculated your answer to part (a)(ii).

Discuss whether (or not) this assumption is likely to be valid.

ASSESSOR'S USE ONLY

(b) In New Zealand, the Ministry of Health recommends that all adults eat at least three servings of vegetables each day. Suppose that the number of servings of vegetables eaten by New Zealand adults each day can be modelled by a Poisson distribution with $\lambda = 3.2$.

(i) Use this Poisson model to calculate an estimate for the proportion of New Zealand adults who eat at least the recommended number of vegetable servings each day.

(ii) What would the value of λ need to be so that, under a Poisson model, at least 95% of New Zealand adults ate at least one serving of vegetables each day?

(iii) Give TWO reasons why the use of the Poisson distribution may not be appropriate to model this situation.

ASSESSOR'S USE ONLY

QUESTION TWO

ASSESSOR'S USE ONLY

(a) It has been estimated that breakfast is eaten daily by 49.9% of New Zealand youths (people aged 15–18 years).

(i) Suppose that a random sample of 8 New Zealand youths was taken.

Using an appropriate probability distribution model, calculate an estimate for the probability that less than half of these 8 youths ate breakfast daily.

(ii) A Year 13 student says, "approximately 50% of youths eat breakfast daily, so 5 of the next 10 youths sampled will eat breakfast daily."

Comment on whether this student is correct in this reasoning.

(iii) For New Zealand children (aged 2–14 years), the proportion who eat breakfast daily is estimated to be 85%.

A health worker observed that for groups of 10 youths, the variation in the number of youths who ate breakfast daily was much greater than the variation they observed for groups of 10 children.

Comment on whether the health worker's observation can be justified statistically.

Support your answer with statistical calculations.

ASSESSOR'S USE ONLY

(b) Suppose the energy intake at breakfast for New Zealand male youths who eat breakfast daily is modelled by a normal distribution, with a mean of 11 200 kJ and standard deviation of 2230 kJ.

(i) Energy intake at breakfast above 12 800 kJ often leads to overeating for the rest of the day.

Estimate the probability that a randomly chosen male youth who eats breakfast daily has an energy intake above 12 800 kJ at breakfast.

(ii) It is known that approximately half of New Zealand male youths who eat breakfast daily have an energy intake at breakfast of between 9500 and 12 000 kJ.

Discuss whether the normal distribution model presented above appears to be appropriate for modelling the energy intake at breakfast of New Zealand male youths who eat breakfast daily.

Support your answer with statistical calculations.

ASSESSOR'S USE ONLY

(iii) Data is collected from students enrolled at a single-sex boys' school. At this school, 15% of youths who eat breakfast daily have an energy intake above 12 800 kJ at breakfast.

Suggest suitable values of the parameters of a normal distribution model that could be used for modelling energy intake at breakfast for male youths who eat breakfast daily at this particular school.

Support your answer with statistical calculations and reasoning.

QUESTION THREE

ASSESSOR'S USE ONLY

(a) A common injury for sports players, which often requires surgery, is a ruptured anterior cruciate ligament (ACL). ACL surgery usually takes between 120 and 150 minutes, with the most common time being 130 minutes.

(i) Using an appropriate probability distribution, calculate an estimate for the probability that an ACL surgery takes less than 130 minutes.

Show working to support your answer.

(ii) Given that an ACL surgery takes less than 140 minutes, calculate an estimate for the probability that the surgery takes more than 130 minutes.

ASSESSOR'S USE ONLY

(b) A hospital is considering sourcing a new x-ray machine. The table below shows the probability distribution of the random variable N, the number of repairs needed for an x-ray machine over any three-year period.

n	0	1	2	3
$P(N = n)$	0.11	0.34	0.35	0.2

(i) Calculate the mean and standard deviation of the number of repairs over the three-year period.

Mean:

Standard deviation:

(ii) The hospital is considering leasing an x-ray machine from Company A or Company B.
Company A charges a one-off fee of $69 500, plus an additional $350 per month.
Company B charges a one-off fee of $65 000, plus an additional $10 000 per repair.

Discuss which company has the higher variation in total lease costs over the three-year period.

Question Three continues on the following page.

(iii) The hospital will lease an x-ray machine for three years.

From which company would you recommend the hospital lease the x-ray machine? Support your answer with statistical reasoning.

ASSESSOR'S USE ONLY

SPARE DIAGRAM

ASSESSOR'S USE ONLY

If you need to redraw your probability distribution models from Question One (a)(i), draw them on the grid below. Make sure it is clear which answer you want marked.

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

91586