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Level 3 Mathematics and Statistics (Statistics) 2021

91586 Apply probability distributions in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (▨). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

It is estimated that on Memorial Avenue in Christchurch there is probability of 0.45 of being required to stop (encountering an amber or red light) at each of the 6 sets of traffic lights on the road. Distances between each set of traffic lights range from 0.6 km at the shortest to 1.2 km at the longest.

(a) Suppose that the number of times a driver is required to stop at the 6 sets of traffic lights when they drive the complete length of Memorial Avenue can be modelled by a binomial distribution.

(i) Justify the use of the binomial distribution to model the number of times a driver is required to stop when they drive the complete length of Memorial Avenue.

(ii) Calculate an estimate for the probability that the driver is not required to stop at any of the 6 sets of lights OR is required to stop at all of the 6 sets of lights when driving the complete length of Memorial Avenue.

(b) For any given intersection with traffic lights on Memorial Avenue, proposed safety changes will increase the probability of being required to stop from 0.45 to 0.6. A driver can be considered 'unlucky' if they are required to stop at more than half of the 6 sets of traffic lights when they drive the complete length of Memorial Avenue.

(i) Calculate an estimate for the probability that a driver will be 'unlucky' when driving the complete length of Memorial Avenue with the proposed safety changes.

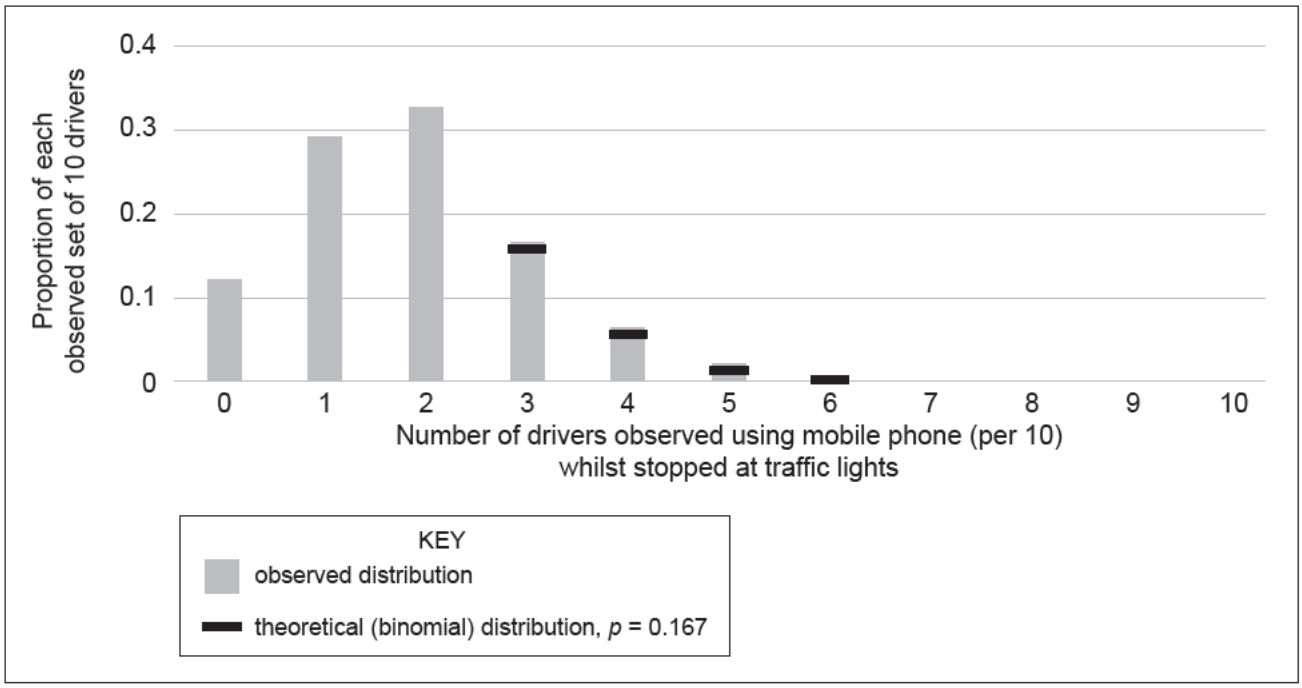
(ii) A person drives the entire length of Memorial Avenue to work and back home over a working week (Monday to Friday inclusive).

Investigate the claim that this person will not experience an increased likelihood of being 'unlucky' on every trip they take along Memorial Avenue during their working week, if the proposed safety changes are put in place.

(c) A recent New Zealand study asked drivers to respond anonymously about their mobile phone habits whilst driving. 16.7% of people admitted to using their mobile phone while stopped at a traffic light. A Canterbury researcher was interested to see if the behaviour of drivers on Memorial Avenue was similar to the drivers in this New Zealand study.

In September 2021, data was collected from groups of 10 drivers stopped at traffic lights on Memorial Avenue. The number of drivers (out of 10) who were using their mobile phones whilst stopped at traffic lights was recorded. The results are shown in the graph below (the experimental distribution shown in grey). These results are compared to the binomial distribution model (the theoretical distribution shown in black).

(i) The graph below shows the observed distribution (grey bars) and a binomial distribution with $p = 0.167$, the theoretical model distribution (shown in black).



Complete the graph by showing the remaining values for the theoretical binomial distribution model.

If you need to redraw your response, use the graph on page 13.

(ii) Discuss TWO features of the observed and theoretical distributions that justify the suitability of this binomial distribution model for the number of drivers (out of 10) observed using their mobile phone.

QUESTION TWO

Wilding pine trees are a serious threat to the natural landscapes of New Zealand. A wilding pine is one that has grown from a seed that was not deliberately planted, and grows in a place where it is not wanted. Research has shown that on Rangitoto Island, the average number of wilding pines per hectare is 1.8.

Adapted from: https://www.google.com/maps

(a) (i) Use a suitable probability distribution to calculate an estimate for the probability that there are more than 4 wilding pines in any hectare on Rangitoto Island.

(ii) Conservationists consider it a success when the probability of finding at least one wilding pine in a hectare is one-third.

Calculate the model parameter that provides this result.

(iii) Rangitoto has an area of 2311 hectares. 100 hectares make up an area of 1 square kilometre.

Provide TWO reasons why it may not be appropriate to use a Poisson model for the number of wilding pines on Rangitoto Island in any square kilometre.

Reason One: _____

Reason Two: _____

(b) Over many years of trying to eliminate wilding pines from New Zealand, it is known that in cleared areas there is a probability of them returning. For the North Island, data suggests this is likely to occur anywhere between 3 and 18 months after the land was cleared. For the South Island, it is likely to be anywhere between 3 and 27 months, with the most likely time at 12 months after the land was cleared.

If wilding pines re-establish themselves within 8 months of an area being cleared, it is expensive to clear the area again.

(i) On the grid below, sketch the two probability distribution models for the time for the wilding pines to return for the North Island and the South Island.

Clearly identify which is the North Island and which is the South Island.

If you need to redraw your models, use the grid on page 13.

(ii) Calculate an estimate for the probability that a randomly selected area in the North Island and a randomly selected area in the South Island both see a re-emergence of wilding pines before 8 months after clearing.

(iii) Calculating the probability in (b)(ii) required at least one assumption to be made.

Explain whether this assumption is likely (or unlikely) to be valid.

QUESTION THREE

Two different computer operating systems, System A and System W, are very popular.

(a) The table below shows the probability distributions of the random variable E, the number of extras (such as speakers or wireless keyboards) an owner of System A and an owner of System W buys for their computer over the time they own it.

e	0	1	2	3	4	5	6	7
System A $P(E = e)$	0.01	0.04	0.10	0.39	0.32	0.08	0.04	0.02
System W $P(E = e)$	0.23	0.12	0.09	0.04	0.05	0.18	0.14	0.15

The expected values for the number of extras for System A and System W are very similar, but the standard deviation for System W is much larger than System A (see values below).

System A: E(E) = 3.47 SD(E) = 1.2
System W: E(E) = 3.41 SD(E) = 2.6

(i) Use statistical reasoning to explain what features of the distributions cause these similarities and differences in expected values and standard deviations for the number of extras purchased.

Reason(s) for similar expected values: _____

Reason(s) for different standard deviations: _____

(ii) Compare the probabilities of buying 5 or more extras for System A and System W.

(b) For both system types, it is believed that the time taken for a serious system error to occur (from purchase) can be modelled by a normal distribution with a mean of 5.5 years and a standard deviation of 0.9 years.

(i) Estimate the probability that, given a computer from either system lasts for more than 6 years until its first serious error, it will last more than 7½ years.

(ii) Another operating system, System L, is known to experience a serious system error within the first 2½ years after purchase or more than 8½ years after purchase, with a total probability of just over 4%.

Justify whether the normal distribution parameters from (b)(i) (mean = 5.5 years, standard deviation = 0.9 years) can be used to model the time to serious error for System L.

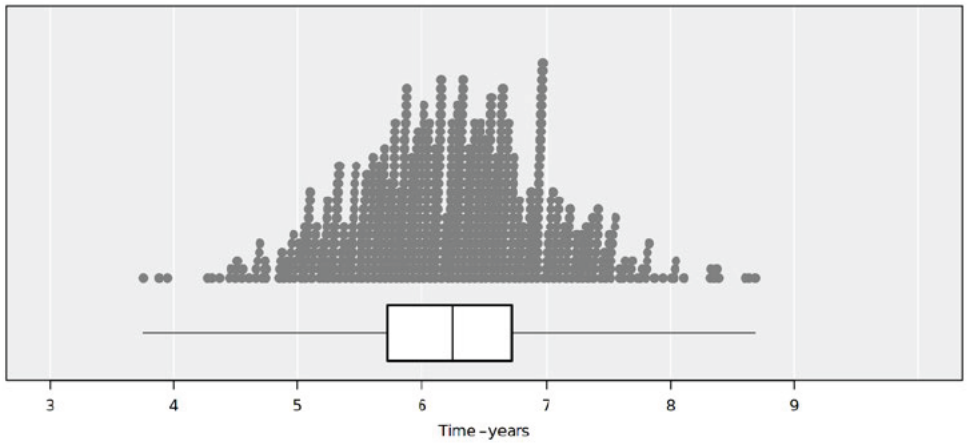
Support your answer with statistical reasoning, including calculations.

Question Three continues on the next page.

(c) Another company developed an operating system, named System C. The company is fairly sure that they have developed System C so that the time for a serious system error to occur is longer on average than the other operating systems (Systems A, W, and L). For System C, the time for a serious system error to occur is on average 6.25 years with standard deviation 0.8 years.

Details of a sample of 800 computers that develop a serious error are sent to the company.

The distribution and summary statistics for the time to serious system error after purchase for this sample are shown in the graph below.



Min	25%	Median	75%	Max	Mean	SD	Sample Size
3.754	5.718	6.246	6.722	8.688	6.236	0.7755	800

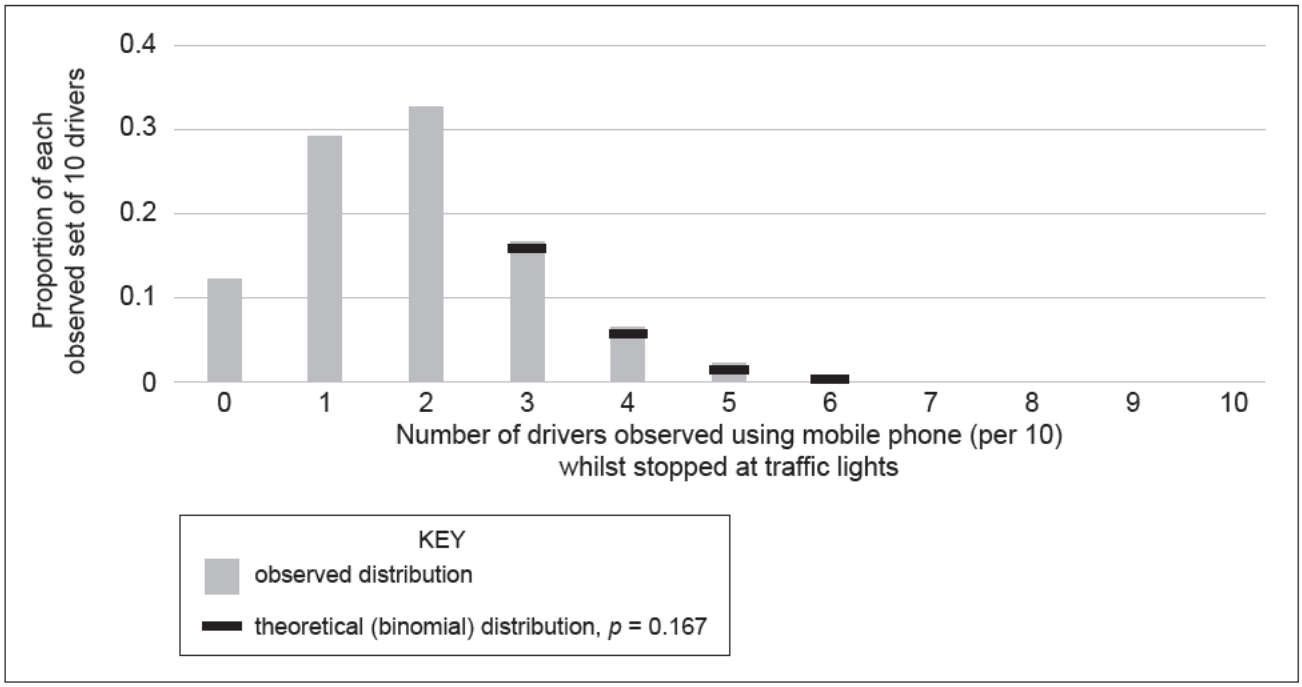
Time to serious error for 800 computers using System C

Justify the use of a normal distribution with mean 6.25 years and standard deviation 0.8 years to model the time to serious error after purchase for System C.

Use statistical reasoning calculations to justify your answer.

SPARE DIAGRAMS

If you need to redraw your response to Question One (c)(i), use the grid below. Make sure it is clear which answer you want marked.



If you need to redraw your response to Question Two (b)(i), use the grid below. Make sure it is clear which answer you want marked.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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QUESTION NUMBER