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Level 3 Mathematics and Statistics (Statistics) 2022

91586 Apply probability distributions in solving problems

Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability distributions in solving problems.	Apply probability distributions, using relational thinking, in solving problems.	Apply probability distributions, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2 – 12 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (▨). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

It has been estimated that for a New Zealand clothing retail company, a purchase is made by 35% of customers entering their stores.

(a) (i) Using an appropriate probability distribution model, calculate an estimate for the probability that less than 3 of the next 10 shoppers make a purchase after entering a store.

(ii) Give TWO assumptions made for the probability distribution model that you used in part (i) to model the number of customers making a purchase when entering a store.

1.

2.

(iii) The clothing retail company has many stores throughout New Zealand.

Using an appropriate probability distribution model, calculate an estimate for the probability that at least 3 out of the next ten customers make a purchase in each of 4 different stores.

(iv) An assumption was made when calculating the probability estimate in part (iii).

Justify this assumption in context.

(b) A particular store offers two alternative online ordering methods for items that are out of stock in the store – 'click and collect' orders and 'home delivery' orders. The amount of time until a customer's 'click and collect' order is ready for collection ranges between 24 hours and 144 hours. For home delivery orders, the time for the order to be delivered can be modelled with a normal distribution, with mean 96 hours and standard deviation 38.4 hours.

Two friends make online orders at the same time, one using the 'click and collect' method, and one using the 'home delivery' method.

Using appropriate probability model(s), estimate the probability that both friends wait more than 120 hours to receive their orders.

Justify any assumptions you have made when estimating this probability.

QUESTION TWO

(a) Management of the retail company have set sales targets of customers purchasing at least 4 items of clothing per 20 minutes in the store. Suppose that the number of items purchased by this company's customers per 20 minutes in the store can be modelled by a Poisson distribution with $\lambda = 2.8$.

(i) Use this Poisson model to calculate an estimate for the probability that in any 20 minute interval a customer purchases less than 4 items of clothing.

(ii) Provide TWO reasons why the use of the Poisson distribution may not be appropriate to model this situation.

Reason One:

Reason Two:

(iii) What would the value of lambda (λ) need to be so that, under a Poisson model, at least 99% of this company's customers purchase at least 1 item of clothing in a 20-minute interval?

(b) The company receives their stock in boxes of 50 clothing items. The items are checked for defects before being sold. The table below shows the probability distribution of the random variable D, the number of defective garments found per box of 50 clothing items received by the company.

d	0	1	2	3
Supplier A P(D = d)	0.73	0.16	0.09	0.02
Supplier B P(D = d)	0.66	0.23	0.07	0.04

(i) Calculate the mean and standard deviation of the number of defects per box of 50 clothing items received by the company from Supplier A.

Mean = _____

Standard deviation = _____

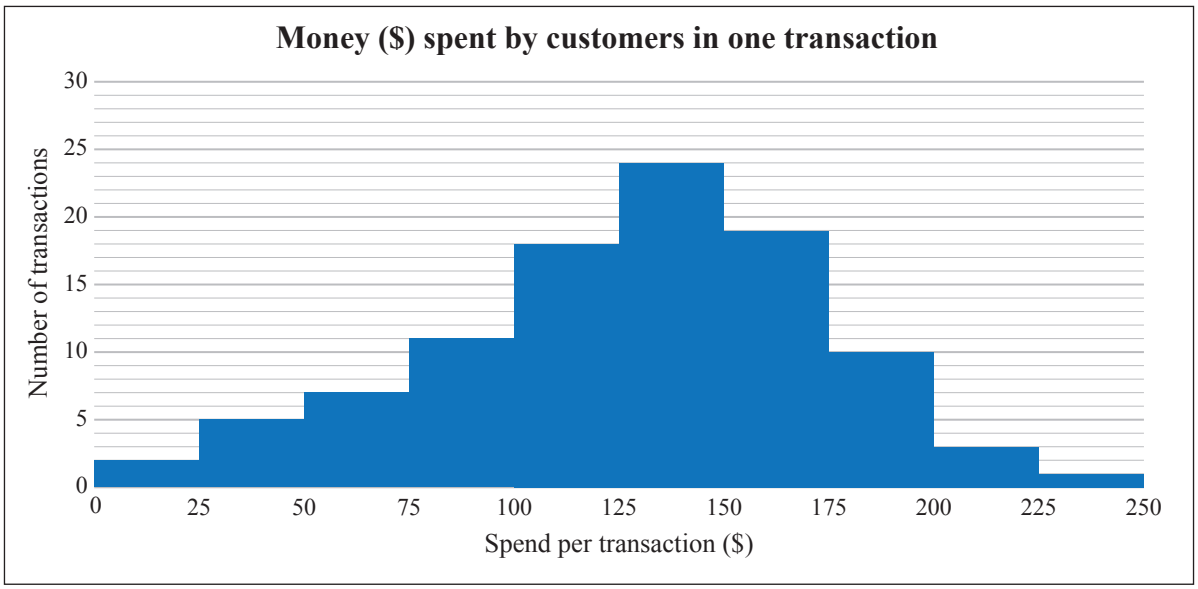
(ii) The clothing retail company prefers to receive boxes of garments with low numbers of defective items. It is also helpful if the number of defective items per box is consistent over multiple boxes.

Use statistical reasoning to explain which supplier fulfils these requirements.

QUESTION THREE

The retail company collects data on the amount of money ($) spent by each customer in one visit to a store (transaction).

Summary statistics:	
Number of transactions	100
Mean spend per transaction	$127.75
Standard deviation of spend	$46.22
Most common spend	$130
Maximum spend	$250



(a) It is claimed that over 25% of transactions consist of spends of more than $150.

Is this claim correct?

Use a calculation to support your answer.

(b) Use a normal distribution model to estimate the probability that a randomly chosen customer spends more than $150 in one transaction.

Show working to support your answer.

(c) Suggest an alternative model that could be used for modelling customer spend per transaction at this particular retail company.

State the parameters of your alternative model.

(d) Use this alternative model to estimate the probability that a randomly chosen customer spends more than $150 in one transaction, given that they spend more than $130.

Question Three continues on the next page.

(e) Compare your probability estimates using the two different probability models with your probability calculated from the data in part (a).

Which model gives the closest estimate of the probability that a customer spends more than $150 in one transaction?

(f) Recommend which model (normal distribution model or alternative model) should be used for modelling the spend per transaction in this particular store.

In your answer you should justify your choice of distribution and identify the parameter(s) of this distribution.

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Write the question number(s) if applicable.

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